

Mathematical Methods in Physics. Program

- Gaussian integrals
 - Gaussian integrals: single and many variables;
 - Wick theorem
- Cauchy integral
 - Complex numbers and analytic functions. Integral in complex variable.
 - Integrals of analytic functions. Cauchy-Riemann equations.
 - Integral Cauchy formula.
 - Taylor and Laurent expansions.
 - Residues. Calculation of integrals via residues.
 - Boundary values of the Cauchy integrals and *Principal Value* integrals.
- Ordinary differential equations
 - Differential equations of the first order. Cauchy problem.
 - Newton's law and integrating multiplier. Energy integral and quadrature integration. Harmonic oscillator. Integrable system.
 - Linear equations with constant coefficients.
- Laplace transform.
 - Operational method. General notions.
 - Direct and inverse Laplace transform. Properties.
 - Convolution. Borel theorem and Duhamel's integral.
- Special functions.
 - Gamma and Beta functions.
 - Hypergeometric function.
 - Orthogonal polynomials.
 - Trigonometric and elliptic functions.
- Asymptotic methods.
 - Series and asymptotic series.
 - Laplace method and stationary phase.

- Asymptotics of some special functions.
- Distributions.
- Linear operators.
 - Vector spaces. Hilbert space. Linear operators and matrices.
 - Differential operators. Schrodinger's operator. Harmonic oscillator.
 - Resolvents. Green functions.
- Matrix integrals.
 - Matrix and eigenvalue integrals.
 - Orthogonal polynomials for one-matrix model.
 - Solution in large N limit.

References

- [1] M. Lavrentiev and B. Shabat, *Methods of the theory of functions of a complex variable*, exists in Spanish (!?), *Metodos de la Teoria de Funciones de Variable Compleja*.
- [2] G. Pradisi, *Lezioni di metodi matematici della fisica*
- [3] F. Olver, *Special functions*.
- [4] I. Krichever and A. Zabrodin, *Lectures on complex analysis*, NRU HSE Mathematical Physics Program.
- [5] A. Pogrebkov, *Lectures on applied methods of analysis*, NRU HSE Mathematical Physics Program.