RELATIVITY AND COSMOLOGY 1

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Tensor calculus

Vectors and tensors in covariant e contravariant form. Two theorems for recognizing the tensorial properties of an object with n-indeces. Metric and metric tensor. Definitions of flat and Riemannian spaces. Covariant derivative of a vector and Christoffel symbols. Definition of geodesics. Parallel transport of a vector in curved space. Riemann tensors. Bianchi identities.

Gravity as a metric theory

The metric in a rotating system. Proper time and proper length. Geodesic motion. Equivalence principle. Geodesic deviation. Weak field limit. Gravitational redshift. Ricci tensor. Field equation in the vacuum.

The space-time geometry outside a given mass distribution

Isotropic matter distribution. Schwarzschild solution. Birkhoff theorem. Equation of motion of a test particle and first integrals. The classical tests of General Relativity: planetary motion and the advance of the Mercury perihelium; deflection of light and gravitational lenses. Metric singularities. Kruskal coordinates and Schwarzschild black hole. Kerr metric: singularities and frame-dragging.

Space-time geometry in a continuum

Energy momentum tensor. Conservation equation and geodesic motions. Field equation. Extension to a non-vanishing cosmological constant. The FLRW metric in the synchronous gauge. The Friedmann equation with and without cosmological constant. Cosmological models: the Milne model; the Einstein static universe; the de Sitter model; dust cosmological models with and without a cosmological constant. Time evolution of the scale factor and the age of the universe.

Distances in cosmology

Proper distance and comoving distance. Particle horizons. Line of sight distance. Angular diameter distance: comparison between different cosmological models. Luminosity distance: the limit of low redshift and the Hubble constant determination. The age of the universe and relative constraints. High redshift SNe Ia and cosmological parameter determinations. Concordance model and dark energy.

The thermal history of the universe

The Cosmic Microwave Background. COBE measurements of the CMB spectrum. CMB temperature, CMB photon number density and the entropy of the universe. Radiation dominated universe: the time-temperature and the time-energy relations. Annihilation of the electron-positron pairs. The re-heating of the CMB. The temperature of the cosmic neutrino background. Primordial nucleosynthesis: the neutron to proton reaction and the ^{4}He abundance. Constraints on the neutrino family number in the early universe. Deuterium abundance as a measurements of the baryonic content on the universe: constraints on $\Omega_b h^2$. The need of dark matter in the universe: a phenomenological discussion of possible candidates.