BASIC ENGLISH FOR SCIENCE

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All sciences, from Physics to Chemistry, from Biology to Information Technology, use the formal language encrypted within the Mathematical code to enhance and quantify their meaning. Formal languages, indeed, are artifices and abstractions, or conventions, no more than natural languages, scientists use to emphasize and convey their sense. This course aims at pointing out the basic English word order and sentence structure and the levels at which English language is interwoven with the mathematical code in order to enable students to build up and cognitively re-produce an essentially coherent and cohesive English text mainly in the context of scientific discourses.

These notes, far from being exhaustive, point out at providing undergraduate university students of sciences with a guideline emphasizing the main steps in building up simple and coherent scientific texts/discourses in English. Some graphs and texts are used as examples and adapted from specified Internet materials in order to allow the students to better interact with the vast material present in Internet. Anyway, if any sources might result incomplete or neglected do not hesitate to contact us in order to further specify the source.

Prerequisites: levels B1, B1.2

TEXT BUILDING

English language is analytic: it means words follow up an ordered sequence that cannot be changed in order for any language functions to make sense. The basic string is

SUBJECT+VERB+OBJECT (S+V+O)                       (1)

If the object is a direct object, then it will generally precede the other objects, so that more than one object is allowed. A V+S+O sequence is only possible when formulating questions. This string gives the sentence its basic structure, which depends on word order. It can sometimes be changed into an O+S+V sequence in order to emphasize some argument with respect to another (an essentially metaphorical use of language, whose preceding (S+V)+O sequence, anyway, can always be inferred) but, strictly speaking, meaning depends on position. Any permutation disrupts the logical sense and alters the meaning to be transmitted.

If we want to connect two or more sentences, we use connectives. The main connectives are and, but, or, because, so, so that, consequently, therefore, thus and the like. But also anything introducing a clause, such as if, when, as soon as, unless, or a relative pronouns, for instance: which, who, that, whose, where. They refer to either the object or the subject (defining relative clause, or kernel) or add extra information (non-defining relative clause, or non-kernel): they modify the sentences while connecting them. If we call C the generic connective, then the basic string becomes

...+S+V+O+C+S+V+O+C+...                                         (2)

that is, a text. A text is made of some words connected together so that they make sense. A paragraph is made of some strings. Paragraphs are further connected by paragraph connectives, and so on.

The indefinite repetition of this basic string in the English language corresponds to the unit of measurement in science so as it is conveyed by Mathematics, for it is the minimal signifying unit.
which every discourse is built upon. Text building, therefore, is going to be the aim of this course: (de)construct scientific texts/discourses shifting in and out the two semiotic codes (language and Mathematics) with respect to (2).

On a wider level we use texts -that is, ordered strings in the form (2) to define everything we deal with. But otherwise we use similar strings each time we deal with something undefined, or unknown. In other words we use well-known and long-established criteria to infer the unknown, the undefined. To this extent sometimes we change the code -we switch from Language to Mathematics, or vice versa, these being the two pragmatic position-dependent codes we use to represent, embody, measure, quantify, emphasize physical (but also unphysical) reality. In so doing we spin in and out of Language and/or Maths every time we talk life into something physical.

Let's do an example. Consider the finite **LIMIT OF A FUNCTION** and let's say we had a graph like this:

Suppose a hole at whatever point on the line. Is the function “f of x”, formally “f(x)” – that is, the graph we can see, the blue line we visualize – defined at that point? What about its value? In other words, what is the appropriate statement defining such a situation? Actually, our x is undefined: it means we cannot say anything about it for it lacks a domain and a range -that is, we don't know where to place it (on a x-axis level), we don't know how to visualize it (on a y-axis level) –that is, using appropriate statements made of an appropriate syntax and word order so as to define it and talk about it using (English) language.

The Mathematical code helps us to solve the problem raised by an f like that. Using its formalism it says we should call L, or f(c), a point that is corresponded by a generic point c on the x-axis. Then it argues the statement we are searching for is

\[
\lim_{x \to c} f(x) = L
\]

(3)

that is, another way of texting a statement. We read it as follows:

“The limit as x approaches c of f(x) equals L.”

(4)

That is to say:

as x approaches c

(5)
or:

as long as x gets closer and closer to c from the positive and the negative direction

the function’s limit = Subject
equals = Verb
L = Object (direct object)

The clauses (5) and/or (6) are introduced by the modifier as (long as) that connects them to the subject. In so doing it works more or less like a relative in a kernel clause and makes the whole phrase “the limit as x approaches c of f(x)” the subject of the (4).

To solve any problems of ‘existence’, or definition, the Mathematical code introduces the conception of continuity saying that the function is continuous at the point where x=c if the function exists at x=c (that is, if f(c), or L, is a real number), if the limit of the function exists at x=c (that is, the limit as x approaches c of the function f(x) is a real number) and if the two values are equal (that’s what equation (3) says, where L equals f(c), of course).

Hence, if a function has a hole, the three conditions above effectively insist that the hole be filled in with a point to be a continuous function. To define things, to make things real and visible, to fill the gap, this semiotic code uses a syntax like that in (3) –more or less in such a way other codes, say computer programs, use other syntaxes in order to emphasize and convey other meanings.

Of course, there must exist also a corresponding conception of ‘discontinuity’.

Let’s read the following text from http://www.milefoot.com/math/calculus/limits/Continuity06.htm:

“When a function is not continuous at a point, then we can say it is discontinuous at that point. There are several types of behaviors that lead to discontinuities.

A removable discontinuity exists when the limit of the function exists, but one or both of the other two conditions is not met. The graphical feature that results is often colloquially called a hole. The first graph below shows a function whose value at \(x = c\) is not defined. The second graph below shows a function which has both a limit and a value at \(x = c\), but the two values are not equal. This type of function is frequently encountered when trying to find slopes of tangent lines.

An infinite discontinuity exists when one of the one-sided limits of the function is infinite. In other words, \(\lim_{x \to c^+} f(x) = \infty\), or one of the other three varieties of infinite limits. If the two one-sided limits have the same value, then the two-sided limit will also exist. Graphically, this situation corresponds to a vertical asymptote. Many rational functions exhibit this type of behavior.
A finite discontinuity exists when the two-sided limit does not exist, but the two one-sided limits are both finite, yet not equal to each other. The graph of a function having this feature will show a vertical gap between the two branches of the function. The function $f(x) = \frac{|x|}{x}$ has this feature. The graph below is of a generic function with a finite discontinuity.

An oscillating discontinuity exists when the values of the function appear to be approaching two or more values simultaneously. A standard example of this situation is the function $f(x) = \sin\left(\frac{1}{x}\right)$, pictured below.

It is possible to construct functions with even stranger discontinuities. Often, mathematicians will refer to these examples as "pathological", because their behavior can seem very counterintuitive. One such example is the function $f(x) = \{x, \text{ when } x \text{ is rational}; 2, \text{ when } x \text{ is irrational}\}$. This function can be proven to be continuous at exactly one point only”.

On a semantic level, we consider as being continuous something that is connected throughout in space and time, its counterpart –the ‘discontinuous’ - being something lacking continuity in space or time. In both cases we represent both the continuity and the gap using a structured string in the form (1) together with (spatial) prepositions.

GRAMMAR REVIEW. We use the imperative in the form ‘Do’ or ‘Let’s do’ also to suggest anything when dealing with a system and/or its own parts.
EXERCISE. What is a limit? What’s that got to do with (dis)continuity? Answer the question using the imperative and an appropriate word order.

Now, let's go back to statement (3). What does it mean? Well, just take a look at the graph below, which performs, figure out and embodies the so-called “epsilon-delta definition” of a limit:

![Graph](http://en.wikipedia.org/wiki/%28%CE%B5,%CE%B4%29-definition_of_limit)

In other words, this code sustains using its own logical operators, given any ε>0 (any epsilon greater than zero) we can always say that

$$0<|x-c|<\delta$$

where $|x-c|$ is simply another way to say “the distance between $x$ and $c$”. That is, you can give me any distance epsilon from $L$ you want, I can always specify a distance $\delta$ (delta) around $c$.

In other words, as long as you pick an $x$ within the domain interval $(c-\delta, c+\delta)$, I can guarantee you that $f(x)$ is within the range $(L-\varepsilon, L+\varepsilon)$. As long as that $x$ you pick up in the domain interval is greater than zero (it does not show up on top of $c$ because the $f$ is undefined at that point) and less than delta, I can guarantee you that the distance between the corresponding $f(x)$ and the limit point $L$ is less than epsilon. In other words:

$$|f(x)-L|< \varepsilon$$

that is to say, the distance between the graph, the curve $f(x)$, and the point $L$, which in turn depends on the point $c$, or the way we approach it along the x-axis, is less than epsilon. The limit as $x$ approaches $c$ of our function of that same $x$ is that same $L$.

That is why, because of the (7) and the (8), which are consequential (in symbols (7) $\iff$ (8), where $\iff = \text{IFF}$), the limit as $x$ approaches $c$ of our function of that same $x$ is equal to that same $L$.

Actually there is no difference, on a semantic level, between (3) and (4). On a structural level we notice a change of notation and a more concise, straightforward use of language. Statement (3) contains statements (4), (5), (6) and can be rewritten as a (2) string using statements (7) and (8).

The symbolic use of language in (3) being paramount, we can conclude that the Mathematical code here is essentially metaphorical, really more than it'd be in the corresponding Linguistic code. Using (3) AND (4) or (3) OR (4), however (here AND/OR work as logic gates), we have filled the gap, the 'hole' of figure 1; whereas connecting the two epsilon/delta intervals in the linguistic event
we have talked the coordinate space into something deictical, its interpretation being determined in relation of the utterance-act with respect to the time and place at which occurs ((6), for instance).

But otherwise, the conception of a limit implies moving along the entire x-axis from the positive and from the negative direction, that is, from the right-end side and from the left-end side and finding out the corresponding function’s value either on the y-axes, that is, in 2 dimension, or, adding the spatial categories of width and depth to that of length, all over the z-axes.

Let’s estimate the value of the following limits reading the following text from http://tutorial.math.lamar.edu/Classes/CalcI/OneSidedLimits.aspx:

\[
\lim_{t \to 0^+} H(t) \quad \text{and} \quad \lim_{t \to 0^-} H(t) \quad \text{where, } H(t) = \begin{cases} 
0 & \text{if } t < 0 \\
1 & \text{if } t \geq 0
\end{cases}
\]

\[
\lim_{t \to 0^+} H(t) \quad \text{and} \quad \lim_{t \to 0^-} H(t) \quad \text{where, } H(t) = \begin{cases} 
0 & \text{if } t < 0 \\
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\end{cases}
\]

\[
\lim_{t \to 0^+} H(t) \quad \text{and} \quad \lim_{t \to 0^-} H(t) \quad \text{where, } H(t) = \begin{cases} 
0 & \text{if } t < 0 \\
1 & \text{if } t \geq 0
\end{cases}
\]

“To remind us what this function looks like here’s the graph.

```
So, we can see that if we stay to the right of \( t = 0 \) (i.e. \( t > 0 \)) then the function is moving in towards a value of 1 as we get closer and closer to \( t = 0 \), but staying to the right. We can therefore say that the right-handed limit is,

\[
\lim_{t \to 0^+} H(t) = 1
\]

Likewise, if we stay to the left of \( t = 0 \) (i.e. \( t < 0 \)), then the function is moving in towards a value of 0 as we get closer and closer to \( t = 0 \), but staying to the left. Therefore the left-handed limit is,

\[
\lim_{t \to 0^-} H(t) = 0
\]

In this example we do get one-sided limits even though the normal limit itself doesn’t exist.”

Again, given
“from the graph of this function shown below, we can see that both of the one-sided limits suffer the same problem that the normal limit did in the previous section. The function does not settle down to a single number on either side of $t=0$. Therefore, neither the left-handed nor the right-handed limit will exist in this case. So, one-sided limits don’t have to exist just as normal limits aren’t guaranteed to exist.”

Moreover, to make things coherent, we say that there exists a discontinuity if the limit as $x$ approaches a number $c$ from the right-end side of a function is not equal to the limit as $x$ approaches that same number $c$ from the left-end side of the same function. So, a function is continuous IFF the two-sided limits are equal.

We now have material enough build up the following table

<table>
<thead>
<tr>
<th>Logical operator</th>
<th>Linguistic (logical) operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$=$</td>
<td>Equals, is equal to, is the same as, …</td>
</tr>
<tr>
<td>$&gt;$</td>
<td>Greater than, larger than, …</td>
</tr>
<tr>
<td>$&lt;$</td>
<td>Less than, smaller than, …</td>
</tr>
<tr>
<td>$\geq$</td>
<td>Greater than or equal to, …</td>
</tr>
<tr>
<td>$\leq$</td>
<td>Less than or equal to…</td>
</tr>
<tr>
<td>$\Rightarrow$</td>
<td>Consequently, so, so that, as a result of this, therefore, this leading/yielding to, …</td>
</tr>
<tr>
<td>$\iff$</td>
<td>IFF, if and only if</td>
</tr>
<tr>
<td>$</td>
<td>a-b</td>
</tr>
<tr>
<td>$x \to c^+$</td>
<td>$x$ tends to $c$ from the positive direction, (as) $x$ approaches $c$ from the right-end side</td>
</tr>
<tr>
<td>$x \to c^-$</td>
<td>$x$ tends to $c$ from the negative direction, (as) $x$ approaches $c$ from the left-end side</td>
</tr>
</tbody>
</table>
and reflect upon some Mathematical symbols and the corresponding connectives of English
‘natural’ language. We can immediately see that they both have the same function when connected
into a discourse and that they both are conventions.

EXERCISE 1. Imagine to walk along any path/trajectory. When and how is it going to be (dis)con-
tinuous? In what way? Write down a text focusing on the language you need to emphasize the tra-
jectory and to move along it and describe what you see all over it.

EXERCISE 2: build up a text corresponding to the question: “How would I find the delta for an
epsilon of 0.01 for the lim as x→2 of (x²-3)?”

ANSWERS KEY TO EXERCISE 2:

1) Solve like normal then when you have δ defined as a function of ε, you can just plug in 0.01
for ε and you're done.

2) We know the limit as x approaches 2 of x²-3 equals 1 (Lim x→2 of (x²-3) = 1) so that, with
respect to (1), L equals 1, c equals 2 while (or as long as) the direction and extent of the
function of that same x follows up or is being described/depicted/illustrated... by x²-3 -that
is, that same x squared minus 3 unities of measurement or that same x shifted by 3 to the
right end side (or L = 1, a = 2, and f(x) = x²-3).

3) We know 0<|x-2|<δ (that is, the distance from any x to the point 2 along the x-axes is less
than delta and greater than zero) and |(x² - 3) - 1| < ε for some ε > 0 . As a consequence we
can state that |x² - 4| = |x-2||x+2| < ε. It means we have to find some way to control the size
of |x+2| since |x-2| is already being kept small by δ. If we were to assume that δ ≤ 1, which is
a pretty generous assumption, we could conclude that (based upon our initial δ inequality) |
x-2|<1 or x is 1 away from 2 in either direction or 1<x<3. Now, since we're trying to control
|x+2|, let's add 2 to the inequality 3<x+2<5 thus |x+2|<5. Hence, |f(x) - 4| < 5|x-2| if |x-2| < δ
≤ 1

and 5|x-2| < ε if |x-2| < ε/5. Therefore if we take δ = min{1,ε/5} then

|f(x)-4|<5|x-2|< 5 * ε/5 = ε if |x-2|<δ.

Therefore, for your value of ε, δ would be ε/5 = (0.01)/5 = 0.002

Answer 1) shows up a straightforward concise to-the-point way of spinning in and out of language.
That's our own elliptical taken-for-granted layout: the less you talk the more you save words. Using
an essentially metaphorical language we rapidly perform our utterance across space/time
coordinates. Anyway, it implies both the speaker and the receiver are sharing the same experience
-that is, they already know how to move across the y/z spaces and along the x dominant values -that
is, they know who's y, who's x and who's f, and the way they work together to build up and
emphasize the meaning: in other words, they know who's f(x).

As for answer (2) and 3) they describe the way we are dealing with space in a more detailed way, 2)
working as a preface and 3) providing the more specific deictic indicators we need to represent
spatial (and temporal) coordinates and to move along them -that is, to talk them into real with
respect to the utterance-act.

EXERCISE 2. Complete the following text: Prove lim as x → 1 of 3(x-1) / x-1, IFF x is not equal to
1, means prove that given any small enough ε > 0 I can give a δ>0 where as long as |x-1|< δ ...

EXERCISE 3. Use any limit (finite or infinite) and build up a coherent text in the form (2)
including the graph within the text.
LANGUAGE FUNCTIONS

So far we have talked about \( f(x) \). Actually, \( f \) of \( x \) means whatever curve, graph, diagram we are going to represent -that is, whatever \( f \) we is going to be performed on the y-axis, for instance, depends on the \( x \) value. In this way a y-range corresponds to an x-domain value (= the set of all things I can input into the function) and vice versa (inverse function), this correspondence being performed by the \( f \). That is why the \( f \) works as an operator: any \( x \) maps into the corresponding \( y \) through the \( f \). In other words:

\[
y = f(x)
\]
or

\[
f : x \rightarrow y
\]
or

\[
\text{input: } x \rightarrow f \rightarrow \text{output: } f(x)
\]
or

\[
\begin{align*}
\text{y/the curve/the graph/...} & \quad \text{(S)} \\
\text{is/equals/can be read (as)/...} & \quad \text{(V)} \\
\text{a function of } x & \quad \text{(O)}
\end{align*}
\]
or

\[
\begin{align*}
\text{f} & \quad \text{(S)} \\
\text{is} & \quad \text{(V)} \\
\text{an application} & \quad \text{(O -direct object)} \\
\text{from } x \text{ to } y & \quad \text{(O -indirect object)}
\end{align*}
\]
or

\[
\begin{align*}
\text{We send any input } x \text{ into an } f \text{-box and we are given an output in the form } f(x) \\
\end{align*}
\]

We can swap the order to enhance a point with respect to another and add an if clause:

\[
\begin{align*}
\text{Given any input } x, \text{ if we send it into an } f \text{-box we will be given an output in the form of } f(x) \\
\end{align*}
\]
but we can see that the whole string sets out as being unaltered and, and as a result of this, the meaning stays the same. Actually, the phrase before the comma can be inferred as a We are given any input $x$ sequence or it may work as an object with respect to any previously attached string (...$+$O$+$C$+$S$+$V$+$O$+...), in this case the if working as a modifier connecting the two sentences, and thus a C); obviously it can be critically appreciated as a (metaphoric) inversion to enhance the point that the $x$ must be the input, the independent variable. Anyway, we can always infer the $(S+V)+O$ string. The same thing might be done with Cs as starting point, this furtherly approaching the utterance-act to infinity. We never stop talking/writing about the (un)physical world via texts/discourses performed by the Linguistic and/or Mathematical code -that is, through language, whose notations is essentially symbolic but rigorously ordered.

F($x$), then. As a matter of facts almost everything in the world can be read (that's metaphoric!) or, better?, can be thought of as a function of something else. Any examples?

a) I will go on holidays according to the money I earn/if I earn some money

  going on holiday = $y$
  earning money = $x$
  according to, if = $f$

  $y = f(x) \Rightarrow$ going on holiday = $f$(earning money)

b) The Ph of an aqueous solution is the negative logarithm of the hydrogen ion concentration

  Ph$ = -\log[H+] \Rightarrow$ Ph$ = f[H+]$

  and then you plot it on your coordinate axes system using a logarithmic scale and you talk the space into something physically real with respect to the coordinate system, and thus with respect to your own utterance-act coordinates using a string of the type (2).

c) Danny charges $10 per hour for painting walls.

In this statement Danny is considered as being a function of the hours he works, so

  $D(h) = 10h, \quad h \geq 0$ (he can't work negative hours)

  “Dustin charges” input in the box is going to give the hours he works

  That is, the number of hours times 10 (multiplied by 10) is equal to 10h.

  For example:

  $D(1/2) = 10 \times \frac{1}{2} = 5h.$

  where the above expression can be read as

  $D(1/2) = D$(one half) = $D$(one over two) = Dustin of half an hour = Dustin as a function of 1h/2

  Both the domain and the range of this function are of course the non-negative rational numbers.

d) Molly charges $25 per hour for tutoring Chemistry with a minimum charge of $15.
15, \(0 \leq h \leq 3/5\) \(\Rightarrow\) at 25$/h in order to make 15 she's going to work \(15/25 = 3/5\) of an hour

\[ M(h) = \begin{cases} 
25, & h > 3/5 \\
\end{cases} \]

\[ \Rightarrow \text{ for every } h \text{ greater than } 3/5 \text{ she's going to charge } 15\$\]

Domain: non negative rational numbers

\[ \text{Range: rational } \geq 15 \]

If Molly's hours are less than or equal to 3/5 (three fifth or three over five) and greater than or equal to zero she's going to charge 15$ because if she only worked, for example, 1/5 of an hour the bill wouldn't be 25$ since she has a minimum charge of 15: she's going to charge 15 $ up until \(3h/5 = 36\) minutes.

Fair enough! We've just found out that we speak the language of Maths functions –or, Maths functions speak our language! That is, any statement we make can be plotted into a graph with a minimum of a couple of variables. What about functions of functions? Well, that's exactly the same thing. Say I had

\[ f(x) = x^2 + 1 \] which is \(x\) raised to the second power, or \(x\) squared, plus one

\[ g(x) = 2x + f(x-3) \] which is two times (or twice) \(x\) plus \(f\) of \(x\) minus three

\[ h(x) = 5x \] which is five times \(x\)

and I want to figure out \(h(g(3))\) -that is \(h\) of \(g\) of 3. Well, since \(g(3)\) equals 6 plus \(f(0)\) and \(f(0)\) equals 1 then \(g(3)\) will equal 7. Consequently \(h(7) = 35\). Of course

EXERCISE 1. Using the same idea you can easily find out \(M(D(3/4))\) or \(D(M(45))\), for instance, using statements \(c)\) and \(d)\) and write down appropriate \((2)\) strings.

On a linguistic level we talk space and time with respect to the utterance-act. Roughly speaking we “attract” and place space, we build trajectories and time span with respect to the \(I\) and the \(you\) of the (linguistic) event, just to name two pronouns, and the system of Cartesian coordinate we build up with respect to us.

“Spatial relations, in particular, are delineated by prepositions and the relationship between context and language is realized through deictics. Persons belongs to the category of deictics (speaker I shifting to hearer you), but the class includes pronominal and demonstrative systems, adverbs, prefixes, particles, verbs which incorporate deictic relations, verbal inflections, tense. For example, \(this, that, there, now, then, go come, take, bring, -ing\) are all part of this example. Examples of deictic elements are: \(\textbf{Stop it!}; \textbf{Come to my house}, \textbf{Let's go to my house}; \textbf{He's going to university}; \textbf{He's coming} to university; \textbf{See you} on the \textbf{left-hand corner}; \textbf{This is mine}; \textbf{Stop there}. It's too dangerous \(\textbf{over})\) \textbf{here}; \textbf{Come over here}. I want to show \textbf{this} thing I’ve found; Move a little \textbf{to the right}; \textbf{Today} is Friday, \textbf{at last}; She was very young \textbf{then}; \textbf{That's enough}; \textbf{This is yours}; \textbf{Turn left} and \textbf{left again}. Notice that the meaning of nonverbal indices depends directly on their relation to the spatio-temporal context to which they are anchored” (F. Trusso, \textit{Spinning in and out of Language}, Nuova Arnica 2001).

As for the spatial relations delineated by preposition we may focus on “reference object geometry (volumes, surfaces and lines: \textbf{in, on, near, at, inside}; single axis: vertical: \textbf{on top of}, horizontal: \textbf{in front of, in back of, beside, along, across}), figure object geometry (single axis: \textbf{along, across, around}; distributed figure: \textbf{all over, throughout, all along, all around, all across}); relation of region to figure object (Relative distance: interior: \textbf{in, inside, throughout}; contact: \textbf{on, all over}; proximal: \textbf{near, all around}. Direction: vertical: \textbf{over, above, under, below, beneath}; horizontal: side-to-side: \textbf{beside, by, alongside, next to}; front-to-back: \textbf{in front of, ahead of, in back of},
behind, beyond), choice of axis system (inherent: on (the) top of, in front of, ahead of, behind; contextual: on top of, in front of, behind, beyond), trajectories (Earth-oriented: up (north), down (south), on the right-end side (east) on the left-end side (west); operators on regions: To: to, into (= to in), onto (= to on); From: from, from under, from inside)” (in Trusso, quoted).

Thus, in order to build up the entire Calculus system we need to focus on the conception of the limit of a function –that is, place anything along a number-line (our x-axes) and, in order to figure it out, that is, in order to visualize it on a vertical axes, that is, to place it all across a system of coordinate axis that allows us to create a figure, we have to ask “as x approaches that anything from both sides (anything greater than or less than that anything), that is, from the right-end side and from the left-end side, (that is, from both the positive and the negative direction), what does the function approach to?”' That is what allows us to decide whether or not a function has a discontinuity, that is, a gap all over the graph, namely if the limit from the right-end side is not equal to the limit from the left end side: in this case the limit of that ‘per se’ anything (not  or ) does not exist and the function is discontinuous.

In order to define the slope of a line you must refer to the graph, choose some trajectory across the line and say that the slope m is the ratio of the change in the y-axes to the change in the x-axis, that is, the change in y over the change in x, that is, the ratio of how much we rise up and down to how much we run on the left and on the right side, that is, in symbols

\[ \text{slope} = m = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \frac{\uparrow \downarrow}{\leftrightarrow} \]

For more complex curves than a line, the slope changes continuously, so that we need to introduce the conception of derivative of the function in order to still quantify and emphasize its variation. Here the slope is obviously a tangent line, but in order to find it any time with respect to any increment, we still have to say the derivative is the limit as the increment approaches zero of the slope of the secant line –that is, as x gets closer and closer to that anything, the secant line will better approximate the tangent line.

Again, when defining inverse functions first we have to focus on the domain  range mapping, then wonder if there is any way to go back from the range to the domain of the function, if there is another function that would take us back again. The same thing when dealing with even and odd functions. Before formally defining an even function we have to realize if it has a symmetry around the y-axes: if we take what’s going on on the right of the y-axes (on our right-end side of the graph we are face-to-face with) and we can reflect it over the other side axes, then we get the other side of the function. Then we realize it’s even IFF (or ) f(x) = f(-x), etc. In the same way, for the odd functions, we look at what is going on on the right of the y-axes, then we reflect it once over the y-axes and then over the x-axes, or we reflect it over the y-axes and then make it negative, then we can write f(x) = - f(-x), etc.

**EXERCISE.** Build a text about the uses of limits and derivatives in describing any functions using all the prepositions and the spatial relations listed above.

**CONTRASTIVE ANALYSIS**

It might be interesting to compare language and Mathematics in order to better identify some of their structural differences and similarities. Let’s read the following text from http://press.princeton.edu/chapters/gowers/gowers_I_2.pdf:
“Let’s consider the following statements:
(1) 5 is the square root of 25.
(2) 5 is less than 10.
(3) 5 is a prime number.
In the first of these sentences, “is” could be replaced by “equals”: it says that two objects, 5 and the square root of 25, are in fact one and the same object, just as it does in the English sentence “London is the capital of the United Kingdom.” In the second sentence, “is” plays a completely different role. The words “less than 10” form an adjectival phrase, specifying a property that numbers may or may not have, and “is” in this sentence is like “is” in the English sentence “grass is green.” As for the third sentence, the word “is” there means “is an example of”, as it does in the English sentence “Mercury is a planet.” These differences are reflected in the fact that the sentences cease to resemble each other when they are written in a more symbolic way. An obvious way to write (1) is $5 = \sqrt{25}$. As for (2), it would usually be written $5 < 10$, where the symbol $<$ means “is less than”. The third sentence would normally not be written symbolically because the concept of a prime number is not quite basic enough to have universally recognized symbols associated with it. However, it is sometimes useful to do so, and then one must invent a suitable symbol. One way to do it would be to adopt the convention that if $n$ is a positive integer, then $P(n)$ stands for the sentence “$n$ is prime”. Another way, which doesn’t hide the word “is”, is to use the language of sets. If we wish to rewrite sentence (3) symbolically, another way to do it is to define $P$ to be the collection, or set, of all prime numbers.

Then (3) can be rewritten, “5 belongs to the set $P$”. This notion of belonging to a set is sufficiently basic to deserve its own symbol, and the symbol used is $\in$. So a fully symbolic way of writing the sentence is $5 \in P$. The members of a set are usually called its elements, and the symbol $\in$ is usually read “is an element of”. So the “is” of sentence (3) is more like $\in$ than $=$. Although one cannot directly substitute the phrase “is an element of” for “is”, one can do so if one is prepared to modify the rest of the sentence a little. There are three common ways to denote a specific set. One is to list its elements inside curly brackets: $\{2, 3, 5, 7, 11, 13, 17, 19\}$, for example, is the set whose elements are the eight numbers 2, 3, 5, 7, 11, 13, 17 and 19. The majority of sets considered by mathematicians are too large for this to be feasible - indeed, they are often infinite - so a second way to denote sets is to use dots to imply a list that is too long to write down: for example, the expressions $\{1, 2, 3, \ldots, 100\}$ and $\{2, 4, 6, 8, \ldots \}$ represent the set of all positive integers up to 100 and the set of all positive even numbers respectively. A third way, and the way that is most important, is to define a set via a property: an example that shows how this is done is the expression $\{x : x \text{ is prime and } x < 20\}$. To read an expression such as this, one first says, “The set of”, because of the curly brackets. Next, one reads the symbol that occurs before the colon. The colon itself one reads as “such that”. Finally, one reads what comes after the colon, which is the property that determines the elements of the set. In this instance, we end up saying, “The set of $x$ such that $x$ is prime and $x$ is less than 20,” which is in fact equal to the set $\{2, 3, 5, 7, 11, 13, 17, 19\}$ considered earlier. Many sentences of mathematics can be rewritten in set-theoretic terms. For example, sentence (2) earlier could be written as $5 \in \{n : n < 10\}$.

Often there is no point in doing this - as here where it is much easier to write $5 < 10$ - but there are circumstances where it becomes extremely convenient. For example, one of the great advances mathematical ones such as numbers, points in in mathematics was the use of Cartesian coordinates to translate geometry into algebra, and the way this was done was to define geometrical objects as sets of points, where points were themselves defined as pairs or triples of numbers. So, for example, the set $\{(x, y) : x^2 + y^2 = 1\}$ is (or represents) a circle of radius 1 about the origin $(0, 0)$. That is because, by Pythagoras’s theorem, the distance from $(0, 0)$ to $(x, y)$ is $x^2 + y^2$, so the sentence “$x^2$
+ y 2 = 1” can be re-expressed geometrically as “the distance from (0, 0) to (x, y) is 1”. If all we ever cared about was which points were in the circle, then we could make do with sentences such as “x2 + y 2 = 1”, but in geometry one commonly wants to consider the entire circle as a single object (rather than as a multiplicity of points, or as a property that points might have), and then set-theoretic language is indispensable. Anyway, sets allow one to reduce greatly the number of parts of speech that one needs, turning almost all of them into nouns. For example, with the help of the membership symbol ∈ one can do without adjectives, as the translation of “5 is a prime number” (where “prime” functions as an adjective) into “5 ∈ P” has already suggested. This is of course an artificial process - imagine replacing “roses are red” by “roses belong to the set R” - but in this context it is not important for the formal language to be natural and easy to understand. Let us now switch attention from the word “is” to other parts of the sentences (1), (2) and (3), focusing first on the phrase “the square root of” in sentence (1). If we wish to think about such a phrase grammatically then we should analyze what sort of role it plays in a sentence, and the analysis is simple: in virtually any mathematical sentence where the phrase appears, it is followed by the name of a number. If the number is n then this produces the slightly longer phrase, “the square root of n”, which is a noun phrase that again denotes a number and plays a similar grammatical role to one (at least when the number is used in its noun sense rather than its “adjective” sense). For instance, replacing “five” by “the square root of 25” in the sentence “five is less than seven” yields a new sentence, “The square root of 25 is less than seven”, that is still grammatically correct (and true). One of the most basic activities of mathematics is to take a mathematical object and transform it into another one, sometimes of the same kind and sometimes not. “The square root of” transforms numbers into numbers, as do “four plus”, “two times”, “the cosine of” and “the logarithm of”. A non-numerical example is “the center of gravity of”, which transforms geometrical shapes (provided they are not too exotic or complicated to have a center of gravity) into points - meaning that if S stands for a shape, then “the center of gravity of S” stands for a point. A function is, roughly speaking, a mathematical transformation of such a kind. When functions appear in mathematical sentences they do not behave like nouns. (They are more like prepositions). To specify a function one must be careful to specify two sets as well: the domain, which is the set of objects to be transformed (like when it is convenient talking about a series of points rather than a geometrical shape), and the range, which is the set of objects they are allowed to be transformed into. Once one starts to speak formally about functions, it becomes important to specify exactly which objects are to be subjected to the transformation in question, and what sort of objects they are transformed into.”

EXERCISE: Which are the main structural differences and similarities in sentences (1), (2), (3)? Can you write other similar sentences and explain them?
Graphs

Sciences deal with graphs in order to emphasize trends, which show the changes and variations of whatever curve as a function of any quantity moving along the x-axis. A trend has a direction and an extent. But otherwise what scientists are really interested in is the rate of change of a graph, which is represented by the derivative—that is, the degree and speed of change, which focus on how large is a variation and how fast it occurred.

As for the directions trends can be up/downward, horizontal, they may show no changes or frequent changes. After appropriately presenting a graph, we use verbs and nouns to enhance the direction of a trend. Some useful hints may be found at http://oppematerjal.sisekaitse.ee/eppeibur/describing_graphs/:

Presenting a graph

<table>
<thead>
<tr>
<th>Introduction</th>
<th>Topic</th>
<th>Circumstances</th>
</tr>
</thead>
<tbody>
<tr>
<td>This graph shows ...</td>
<td>the results of our products ...</td>
<td>over 10 years.</td>
</tr>
<tr>
<td>The diagram outlines ...</td>
<td>rates of economic growth ...</td>
<td>between 1990 and 1996.</td>
</tr>
<tr>
<td>This table lists ...</td>
<td>the top ten agencies ...</td>
<td>in the industrial world.</td>
</tr>
<tr>
<td>This pie chart represents</td>
<td>the company's turnover ...</td>
<td>for this year in our sector.</td>
</tr>
<tr>
<td>This line chart depicts ...</td>
<td>the changes in sales ...</td>
<td>over the past year.</td>
</tr>
<tr>
<td>This chart breaks down ...</td>
<td>the sales of each salesman ...</td>
<td>during the past ten weeks.</td>
</tr>
</tbody>
</table>

The four basic trends are:

1* upward movement : ↑
2* downward movement : ↓
3* no movement : ⇄
4* change in direction : ↑” or “↓

Indicating upward movement : ↑

<table>
<thead>
<tr>
<th></th>
<th>Transitive</th>
<th>Intransitive</th>
</tr>
</thead>
<tbody>
<tr>
<td>(to) increase</td>
<td>(to) increase</td>
<td>(an) increase</td>
</tr>
<tr>
<td>(to) raise</td>
<td>(to) rise (rose, risen)</td>
<td>(a) raise (US), a rise (UK)</td>
</tr>
<tr>
<td>(to) push/put/step up</td>
<td>(to) go/be up</td>
<td>(an) upswing</td>
</tr>
<tr>
<td>(to) extend, (to) expand</td>
<td>(to) extend, (to) expand</td>
<td>(an) extension, expansion</td>
</tr>
<tr>
<td>(to) progress</td>
<td>(a) progression</td>
<td></td>
</tr>
<tr>
<td>(to) boom/soar/climb</td>
<td>(a) boom</td>
<td></td>
</tr>
<tr>
<td>(to) jump, (to) skyrocket</td>
<td>(a) jump</td>
<td></td>
</tr>
<tr>
<td>(to) reach a peak, (to) peak</td>
<td>(a) peak</td>
<td></td>
</tr>
<tr>
<td>(to) reach an all-time high</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Indicating downward movement : ↓

<table>
<thead>
<tr>
<th>Verbs</th>
<th>Nouns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transitive</td>
<td>Intransitive</td>
</tr>
<tr>
<td>------------</td>
<td>-------------</td>
</tr>
<tr>
<td>(to) decrease</td>
<td>(to) decrease</td>
</tr>
<tr>
<td>(to) cut, (to) reduce</td>
<td>(a) cut, (a) reduction</td>
</tr>
<tr>
<td>(to) fall (off)</td>
<td>(fall, fell, fallen)</td>
</tr>
<tr>
<td>(to) plunge, to plummet</td>
<td>(a) plunge</td>
</tr>
<tr>
<td>(to) drop (off)</td>
<td>(a) drop</td>
</tr>
<tr>
<td>(to) go down</td>
<td>(a) downswing</td>
</tr>
<tr>
<td>(to) decline</td>
<td>(a) decline</td>
</tr>
<tr>
<td>(to) collapse</td>
<td>(a) collapse (dramatic fall)</td>
</tr>
<tr>
<td>(to) slump, (to) go bust</td>
<td>(a) slump</td>
</tr>
<tr>
<td>(to) bottom out</td>
<td></td>
</tr>
</tbody>
</table>

**Indicating no movement**: ☑️

<table>
<thead>
<tr>
<th>Transitive</th>
<th>Intransitive</th>
</tr>
</thead>
<tbody>
<tr>
<td>(to) keep ... stable</td>
<td>(to) remain stable</td>
</tr>
<tr>
<td>(to) hold ... constant</td>
<td>(to) stay constant</td>
</tr>
<tr>
<td>(to) stabilize</td>
<td>(to) stabilize</td>
</tr>
</tbody>
</table>

**Indicating a change of direction**: ⊤ or ⊥ ...

<table>
<thead>
<tr>
<th>Transitive</th>
<th>Intransitive</th>
</tr>
</thead>
<tbody>
<tr>
<td>(to) level off</td>
<td>(to) level off/out, to flatten out</td>
</tr>
<tr>
<td>(to) stop falling/rising</td>
<td>(a) change</td>
</tr>
<tr>
<td>(to) stand at</td>
<td>(to) remain steady</td>
</tr>
<tr>
<td>(to) stop falling and start rising</td>
<td></td>
</tr>
<tr>
<td>(to) stop rising and start falling</td>
<td></td>
</tr>
</tbody>
</table>

**All at once:**
- UP – **Verbs** rise, increase, grow, go up, improve, jump, surge, shoot up, soar, rocket
- UP – **Nouns** a rise, an increase, growth, an upward/rising/increasing trend, an improvement, a jump, a surge
- DOWN – **Verbs** fall, decrease, drop, decline, go down, slump, plummet
- DOWN – **Nouns** a fall, a decrease, a decline, a downward/falling/decreasing trend, a slump
- NO CHANGE – **Verbs** remain stable/constant, stay at the same level, stabilize
- FREQUENT CHANGE – **Verb** fluctuate
- FREQUENT CHANGE – **Noun** – fluctuation
- AT THE TOP – **Verbs** reach a peak, peak., reach its/their highest point
- AT THE BOTTOM – **Verbs** reach/hit a low (point), hit/reach its/their lowest point

**EXERCISE.** Being a chemistry student, use the verbs, noun and phrases above to describe the trend’s direction of the following chart and graph and give them an appropriate title.
EXERCISE. Being a biologist describe the trend’s direction of the following graphs and give them an appropriate title.
EXERCISE. Being an IT's student describe the trend’s direction of the following graphs and give them an appropriate title.
### Time spent on Incidents by Service

- Postini
- Outlook (Mac)
- Wire Networking
- JICS (Portal)
- Wireless Networking
- Personal File, Print, Web Access
- Jenzabar CX
- Special Task Software Support
- Printing Troubleshooting
- Kronos
- Classroom Tech Support
- Laptop, Tablet, Workstation
- Virus/Malware Remediation
- Student Computer Support

![Bar Chart: Time spent on Incidents by Service](chart.png)

### BOP spending on ICT

- **Total Spending:** $51.4 billion

<table>
<thead>
<tr>
<th>Region</th>
<th>$ billions (PPP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td>4.4</td>
</tr>
<tr>
<td>Asia</td>
<td>28.3</td>
</tr>
<tr>
<td>Eastern Europe</td>
<td>5.3</td>
</tr>
<tr>
<td>Latin America</td>
<td>13.4</td>
</tr>
</tbody>
</table>

Each square represents approximately $200 million
Now, focus on

Describing the elements of a graph

Look at the graph and write the appropriate letters in front of each definition:

- : the horizontal axis (or the x axis)
- : the vertical axis (or the y axis)
- : the scale
- : a solid line
- : a broken line
- : a dotted line

Then match them into a coherent 15-line text using all that you have learned so far about graphs.

EXERCISE. Being a Physics student describe the trend’s direction of the following graph and give it an appropriate title.
As for the extent of a trend we may focus on the difference between two points (verb/noun + by + number) or just describe the end point (verb/noun + to + number)

For example
The force has risen by 35 N from 1kg to 8kgs of mass.
There has been a rise of 35N in force.
Likewise, using another graph, you can say that something falls to a certain quantity or that there was a fall in something to a certain quantity.
EXERCISE. Draw a graph that fits the statement above.
EXERCISE Add the information about the extent to the previous graph-description texts.

As for the rate of change we may consider the following scheme.
As a matter of fact in these descriptions adjectives and adverbs are used as nouns’ and verbs’ modifiers:

**Adjectives:**
- dramatic, considerable, sharp, significant, moderate, slight, sudden, rapid, steady, gradual, slow

**Adverbs:**
- dramatically, considerably, sharply, significantly, moderately, slightly, suddenly, rapidly, steadily, gradually, slowly

Now, consider the following types of charts again from [http://oppematerial.sisekaitse.ee/eppleibur/describing_graphs/](http://oppematerial.sisekaitse.ee/eppleibur/describing_graphs/) and write about, apply them to whatever subject and describe, if any, their speed/degree of change. Then do the following exercise.

- **Pie chart**

  A pie chart is used to show percentages

- **Bar chart**
A bar chart is used to compare different sets of information.

- Line graph

A line graph is most useful for showing trends.

The total property crime rate and B&E rate between 2000 and 2007.
EXERCISE: fill in the gaps

1. Introduction
This report examines the changes in the total property crime rate and the break and enter rate between 2000 and 2007.

2. Findings.
In 2000 the total property crime rate was 2500 offences per 100 000 population. Then the rate 1) rose ............ (sharply/sharp) and reached 3800 in 2001. After a 2) ............ (moderate/moderately) fall in 2002, the rate started to 3) ............ (increase/decrease) again and reached a 4) ............ (peak/top) in 2003. However, after 5) ............ (stabilizing/fluctuating) for some months, the total property crime rate dropped 6) ............ (considerably/considerable) throughout 2004 and the beginning of 2005. The rate stayed at about 2400 offences from mid-2005 7) ............ (to/until) mid-2006 before 8) ............ (decreasing/decrease) again.

In 2000 the break and enter rate was about 760 offences per 100 000 population. 9) From (From/since) 2000 to 2003, there was a steady 10) ............ (upward/downward) trend in the B rate. The rate 11) ............ (reached/arrived) its highest point in 2003 and then 12) ............ (showed/fell) a significant downward trend until mid-2005. After being stable for a few months, the rate continued to fall 13) ............ (slightly/slight), dropping to around 780 in 2007.

3. Conclusion
The total property crime rate fluctuated from 2000 to 2003, whereas the break and enter rate showed a general upward trend. Both rates peaked in 2003, fell significantly until mid-2005, stabilized for some months and 14) ............ (fell/grew) slightly during 2006 and 2007.

Now look at the resume, analyze the example and do the following exercises.
A flow chart is a diagram showing the progress of material through the steps of a manufacturing process or the succession of operations in a complex activity.

A pie chart displays the size (of each part as a percentage of a whole (un tout)).

A (vertical or horizontal) bar chart is used to compare unlike (different) items.

A line chart depicts changes over a period of time, showing data and trends.

A table is a convenient way to show large amount of data in a small space.

A diagram is a drawing showing arrangements and situations, such as networks, distribution, fluctuation ...

Analysing an example

The x axis of this graph shows the twelve months of the past year while our sales in millions of dollars appear on the y axis. It may be seen clearly that sales rose steadily in the first half of the year (from January to May) and reached their peak in June. Then they dropped off in July and levelled out in August. After rising sharply during September, they suffered a dramatic fall in October but then made a significant (sensible) recovery in November. However, the year ended with a slight downturn.
1. Match each sentence below with one of the following graphs

1. **a** The investment level rose suddenly.
1. **d** The sales of our products fell slightly in the final quarter.
1. **b** The Research and Development budget has stabilized over the past few years.
1. **d** At the end of the first year, sales stood at 50 per cent of the present level.
1. **c** The price reached a peak before falling a little and then maintaining the same level.
1. **f** There has been a steady increase in costs over several years.
1. **e** The sudden collapse in share prices has surprised everyone.
1. **h** The value of the shares has shown a steady decline.

2. Look at the graph below, then complete the sentences.

1. The ............................................... compares three products: A, B and C.
1. The ............. shows time over ten years while the ............. shows sales in number of units.
1. As you can see, product A is represented by the ............................................
1. The performance of Product B is shown by the ..............................................
1. And a …………………………………. has been used to show the results of Product C.
1. Clearly, ................................... is the most successful product ....................................
1. Sales of Product B ...................... in recent years while sales of Product C ......................
1. On the contrary, product A has shown a .................................................
3. Read the following text and draw the corresponding graph on the right.

The graph opposite covers the years 1976 to 1995. It shows that the number of television viewing hours rose steadily and steeply during that period in the US, starting at just under 5 hours a day to reach more than 7 hours in 1995. There was a slight increase in 1982 and sharper falls in 1986 and 1991. The next decrease, in 1994, is hardly significant. Though we do not have the latest figures, it is unlikely that the trend will have reversed.

4. Comment on the graph below using and organising the following expressions:

Sales rose / went up / increased / climbed ...
+ adverb (slowly / steadily / rapidly / gradually ...)
Sales stood at ...
Sales peaked / peaked out
Sales levelled out / flattened out
Sales bottomed out
This was due to ...
This was the result of ...
This caused ...
This led to ...

Avoid repetitions!

Conclude by saying whether this graph is typical or not; Justify your answer.
THE ACTUAL RATE OF CHANGE: THE DERIVATIVE

Let’s try to find out the slope at any point of the curve f(x) = x^2. We want to calculate f’(x) = the derivative of f(x) = the limit as the increment h approaches zero of the slope of the secant line at that point, that is, \[ \frac{f(x+h) - f(x)}{h} \]. Actually, the limit as h \to 0 is the slope of the tangent line we are searching for, that is the derivative. In our case f’(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{2x + h}{h} = 2x. So, for instance, f’(7) = 14, f’(2) = 4, f’(0) = 0, the latter meaning the slope of the tangent line is exactly the x-axis.

Let’s read the following text, to be found at http://www.teacherschoice.com.au/Maths_Library/Calculus/plot_derivative_graphs.htm:

The 'derivative' or 'derived function' evaluates to the gradient of the original function at any given 'x' value.

Derivative graphs can tell you a lot about a function. The following graph shows the original function (blue) and its derivative (red) plotted together:

Consider the function plotted in blue above:
\[ y = 0.5x^2 - x - 1.5 \]
Using the rules of calculus, the derived function is:
\[
\frac{dy}{dx} = 2 \times 0.5x^{2-1} - 1 \times x^{1-1} - 0
\]
\[= x - 1\]

For any 'x' value, the 'y' value of the derived function equals the gradient of a tangent to the original function at that point. Here are some brief observations from the graph above:
Original function: \[ y = 0.5x^2 - x - 1.5 \]
For 'x' values less than \( x=1 \), the original function (blue) is decreasing left to right, so all tangent lines will have a negative gradient.

At \( x = 1 \), the original function has a turning point where the gradient is zero.

For 'x' values greater than zero, the original function is increasing left to right, so all tangent lines will have a positive gradient.

Derived function: \( \frac{dy}{dx} = x - 1 \)
For 'x' values less than \( x=1 \), the derived function (red) is negative.

At \( x = 1 \), the derived function is zero.

For 'x' values greater than \( x = 1 \), the derived function is positive.

Also, the derived function 'x - 1' is a straight line, which indicates that the gradient of the original function increases at a constant rate as 'x' increases.

**EXERCISE:** plot any functions and its derivative and build up a coherent text to describe it.

**RATE OF CHANGE PROBLEM.** Consider the following scenario: you have a reversed cone like a cup whose both height and diameter of the top of the cup are 4 cm and you are pouring water into this cup at a rate of 1 cubic centimeter per second (1cm³/sec), and right at this moment there is a height of 2 cm of water in the cup. So, right at this moment, what is the rate at which the height of the water is changing? -that is, how fast is it changing? Solve the problem building up a coherent English text and appropriately fit the numerals within the text.

**BASIC LOGICAL FUNCTIONS**

A logical connective or operator (see [http://en.wikipedia.org/wiki/Logical_connective](http://en.wikipedia.org/wiki/Logical_connective)) is a symbol or word used to connect two or more sentences such that the sense of the compound sentence produced depends only on the original sentences. The most common logical connectives are binary connectives, which join two sentence. Negation is considered to be an unary connective. English language, in particular, is analytic: it means words follow up an ordered sequence. The basic string is SUBJECT+VERB+OBJECT (S+V+O). Any permutation disrupts the logical sense and alters the meaning to be transmitted. If we want to connect two or more sentences, we use connectives.
Various English words and word pairs express logical connectives. Examples are: "and" (Logical conjunction), "and then" (Logical conjunction with sequencing), "and then within" (Logical conjunction with sequencing and time window requirement), "or" (Logical disjunction), "either...or" (Exclusive or/exclusive disjunction), "implies" (Material conditional/implication), "if...then" (Material conditional/implication), "if and only if" (logical biconditional/equivalence), "only if" (logical biconditional/equivalence), "just in case" (logical biconditional/equivalence), "but" (logical conjunction), "however" (Logical conjunction), "not both" (Sheffer stroke/alternative denial), "neither...nor" (Logical NOR/joint denial). The word "not" (negation) and the phrases "it is false that" (negation) and "it is not the case that" (negation) also express a logical connective – even though they are applied to a single statement, and do not connect two statements.

Digital logic is a rational process for making simple true or false decisions based on the rules of Boolean algebra. True can be represented by a 1 and false by a 0, so that in logic circuits the numerals appear as signals of 2 different voltages (or currents, frequencies, etc). Often in lab work it's helpful to use a LED to show when a signal is 0 or 1. Usually a 1 is indicated with a LED that is ON (i.e. glowing). You can use the buttons below to check out this AND gate (Note what an AND gate symbol looks like!) with a simulated LED. Note the following in the simulation (and you can use this in your lab experiments).

- To get a logical zero, connect the input of the gate to ground to have zero (0) volts input.
- To get a logical one, connect the input of the gate to a five (5) volts source to have five volts at the input.
- Each button controls one switch (two buttons - two switches) so that you can control the individual inputs to the gate.

Each time you click a button, you toggle the switch to the opposite position.

Computers look at a list of inputs and produce an output. We use TRUTH TABLES to show this list of inputs and outputs. The simplest one is for one input and one output (compare to Avezzano Comes F., Plug In, Hoepli 2002):

<table>
<thead>
<tr>
<th>TABLE A</th>
<th>TABLE B</th>
</tr>
</thead>
<tbody>
<tr>
<td>INPUT</td>
<td>OUTPUT</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

For table A it is the same as having a wire connecting the input and the output: whatever appears on the input is transferred to the output. For table B the opposite happens. This is known as a NOT operation: the output is 'not' the input. The output will be true (1) IF (or WHEN) the input is false (0). The NOT operation is represented as not A=Ā (that is, Ā= ¬A). Actually, the NOT is a unary operator on a proposition Q. ¬Q means not Q, the logical opposite (negation) of Q. The effect of the unary operator ¬ is to reverse the truth value of the statement following it.

Another way to look at a truth table is as a list of possible events which states what will happen in each case. Suppose your friend John might be coming to visit. If he comes then you won't (= will not) play computer games. If he doesn't come you will play computer games. This is an example of IF CLAUSE 1. We can also use IF CLAUSES 2 or 3. In all cases the corresponding truth table would look like the following:

<table>
<thead>
<tr>
<th>EVENT</th>
<th>RESULT</th>
</tr>
</thead>
<tbody>
<tr>
<td>(if) John comes to visit</td>
<td>(then) you won't play computer g.</td>
</tr>
</tbody>
</table>
(if) John doesn’t come to visit (then) you will play computer g.

There is an obvious correspondence between an if clause/event/input and a (main)clause/result/output. Main/if clauses are known in linguistics as kernel/non kernel clauses -the corresponding Kernel in the Maths code being an \( m \times n \) matrix \( A \) with coefficients in a field \( K \). Actually it is the set

\[
\{ \mathbf{0} \}
\]

where \( \mathbf{0} \) denotes the zero vector with \( m \) components. The matrix equation \( Ax=0 \) is equivalent to a homogeneous system of linear equations. The kernel being the solution set to such a system (in \( \mathbb{R}^3 \), for instance, a line through the origin) it obviously deals with a defining clause rather than a non-defining one, the main proposition/clause (that is, the output) defining the sentence in order for it to make sense. A set of solution is actually an output, that is, the result of any (linguistic) event. Anyway, we are going to better discuss about that when dealing with relative clauses.

As for the three if clauses we can write

1) If John comes to visit, then you won’t play computer games
2) If John came to visit, then you wouldn’t play computer games
3) If John hadn’t come to visit, then you wouldn’t have played computer g.

A gate can have one, two or more inputs and a single output. Let's consider a device that performs an AND operation. If the inputs \( A \) and \( B \) are both True (1) then the output \( C \) will also be True (1). Otherwise \( C \) will be false. We use an AND device to perform a binary operation on two propositions (clauses): \( P \land Q \) (\( P \cdot Q \)) means \( P \) is true and \( Q \) is also true. Using Boolean algebra we have \( C = A \cdot B \) or \( C = AB \).

The truth table would look like this:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Another possibility for a device with two inputs and one output is an OR operation: if \( A \) or \( B \) is True (1) then \( C \) is also True (1). Otherwise \( C \) is false. We also use an OR device to perform a binary operation on two propositions (clauses): \( P \lor Q \) (\( P + Q \)) means either \( P \) is true or \( Q \) is true, or both. An OR gate is a gate for which the output is 1 whenever one or more of the inputs is 1. The output of an OR gate is 0 only when all inputs are 0. In terms of Boolean variables, \( C = A + B \):

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Let's study this example from Plug In (quoted): Andy A and Bruce B both like the cinema, but they don't like each other. As a result Andy will not go to the cinema \( C \) if Bruce goes. Feelings are mutual, and Bruce will not go if Andy goes. Show Boolean equations for: a)somebody going to the cinema; and b)nobody going to the cinema.
Solution: the two variables are Andy and Bruce. There are 4 possible combinations: 1) neither Andy nor Bruce goes; 2) Andy does not go but Bruce does; 3) Andy goes but Bruce does not; 4) both Andy and Bruce go.

We represent going to the cinema by a 1 and not going by a 0. of the four possibilities only 2 result in anyone going to the cinema:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

The possibilities for somebody going to the cinema have a 1 in the C column. These are:

\[ \text{NOT Andy (Ā) AND Bruce (B)} \]
\[ \text{OR} \]
\[ \{ C = ĀB + AB \} \]

\[ \text{Andy (A) AND NOT Bruce (B)} \]

To conclude just take a look at some more gates. First the NAND, which is again a binary operation on 2 propositions: \( P \uparrow Q \) means not \( P \) and \( Q \) together, so that it is an and gate with an inverter on the output. Also the NOR is a binary operation on 2 propositions. Anyway, here \( P \downarrow Q \) means neither \( P \) nor \( Q \) (just like in case 1) of our Andy-Bruce-cinema situation).

EXERCISES

Construct appropriate truth tables, write the corresponding Boolean equations and write appropriate if clauses 1, 2, 3 for each numeral.

1) Let’s consider a case where a pressure can be high and a temperature can be high. Let’s assume we have two sensors that measure temperature and pressure. The first sensor has an output, \( T \), that is 1 when a temperature in a boiler is too high, and 0 otherwise. The second sensor produces an output, \( P \), that is 1 when the pressure is too high, and 0 otherwise. Now, for the boiler, we have a dangerous situation when either the temperature or the pressure is too high. It only takes one. Construct a truth table for this situation. The output, \( D \), is 1 when danger exists.

2) The alarm will sound if it senses heat or smoke.

3) If a two-carbon acetyl group is transferred from acetyl-CoA (Acetyl coenzyme A) to the four-carbon acceptor compound (oxaloacetate) to form a six-carbon compound (citrate) and this citrate goes through
a series of chemical transformations, losing two carboxyl groups as CO$_2$, then the citric acid cycle begins.

4) If the hydrogen ion concentration in water is ten raised to the negative seventh power and either the PH is defined as the negative logarithm of the hydrogen ion concentration or the hydrogen ion concentration is defined as 10 raised to the negative PH, then the PH of water is equal to 7.

\[10^{-\text{PH}} = [H^+] \iff \text{pH} = -\log_{10} [H^+] = -\log (10^{-7}) = (-7) = 7]\]

5) If a system of $N$ particles, $P_i$, $i=1,...,N$, are assembled into a rigid body and $F_i$ is the external force applied to particle $P_i$ with mass $m_i$ then Newton's second law can be applied to each of the particles in the body:

\[
F = m \cdot a
\]

where $F_{ij}$ is the internal force of particle $P_j$ acting on particle $P_i$ that maintains the constant distance between these particles.

6) You need to control two pumps that supply two different concentrations of reactant to a chemical process. The strong reactant is used when pH is very far from the desired value, and the weak reactant when pH is close to desired. You need to ensure that only one of the two pumps runs at any time. (Each pump controller responds to standard logic signals, that is when the input to the pump controller is 1, the pump operates, and when that input is 0 the pump does not operate).

Example Solution exercise 6) from http://www.facstaff.bucknell.edu/mastascu/elessonshtml/Logic/Logic1.html. Let's look at this problem with a truth table. Here's the truth table.

<table>
<thead>
<tr>
<th>Pumps On</th>
<th>Pump Choice</th>
<th>Pumps S</th>
<th>Pump W</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1 0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
In English, we say to turn Pump S (Strong reactant) ON when the pumps are ON, and the strong reactant is chosen (Choice 3) and to turn Pump W (Weak reactant) ON when the pumps are ON and the weak reactant is chosen (Choice 2). Otherwise, do nothing.

If we examine it closely we see that there is exactly one term in each function. S is 1 only for choice 3, that is when you want PUMPS ON and you want the strong reactant. Similarly, W is 1 only for choice 2. Here's the truth table again. Note the following:

- We have defined Boolean variables here for the various signals, P, C, S, and W.
- We have indicated the inputs by shading them green, and the outputs by shading them orange.

<table>
<thead>
<tr>
<th>P</th>
<th>C</th>
<th>S</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Looking at the statement "S is 1 . . . when you want Pumps ON AND you want the strong reactant" then you can generate a logic expression directly from the statement.

and also:

Finally, realize that it doesn't take much to implement these functions. Note you only need one inverter and two AND gates. Here's the circuit that turns the pumps on at the proper time. This is an interactive simulation of the circuit, so you can toggle the switches with the push buttons. Check it out.

**TENSES: PRESENT CONSOLIDATION**

Scientific subjects deal with working systems. Actually, to simply describe systems or state something about them, or tell about the way they usually work we use the simple present tense. But as soon as the system is at work, whatever system, we use the present continuous.

On a grammar level it makes sense, for the present simple places statements, truth universally acknowledged or repeated actions across time using time adverbs emphasizing such a context. They are every time/day/night, etc. (that is, every + time expression), once/twice/n times a week/day, etc., in/within n day(s)/month(s), etc., frequency adverbs like usually, sometimes, always, often, never, ever, hardly ever.
The present continuous, on the other side, emphasizes the time of speaking with respect to adverbs like now, at the moment, today, this/these year/days/months, etc.

To build up all the English verbs we use the verbal paradigms –that is, base form, past simple and past participle, depending on whether or not they be regular or irregular verbs.

For the present simple we use the following scheme:
Positive statements:  S + base form of the paradigms (+ “s” 3rd person)
Negative: S + don’t/doesn’t + base form
Interrogatives: S + do/does + s + base form.

For the present continuous we need to conjugate the verb be and use an -ing form as follows:
positive  ⟷ S + am/is/are + ing form
negative  ⟷ S + am/is/are + not + -ing form
interrogative  ⟷ am/is/are + S + - ing form.

As for the present simple passive (remembering that the object of the active becomes the subject of the passive, that the agent must be preceded by the preposition “by” and specified only if the subject is specified (neither a pronoun nor a someone or nobody) and that there are verbs followed by two objects such as, for instance, ask, give, send, say, tell, teach, pay, offer for which the indirect object is preceded by the direct object), the basic string is S+ am/is/are + past participle, while the present continuous passive is like S + am/is/are + being + past participle. Let’s contextualize it into the following:

**TALKING THE PRESENT INTO A REDOX REACTION**

Redox is an acronym that stands for reduction/oxidation. During a chemical reaction, or equation, some reactants are being transformed into some products. We generally associate an oxidation state to the charge an atom would have if all bonds to atoms of different elements were 100% ionic. Thus the oxidation number is connected to the charge. Let's consider, for instance, the molecule of sodium chloride. We know that sodium is an alkaline metal and that it has one valence electron in group I, while chlorine is an halogen of group VII that just needs 1 electron to have full 8 valence electrons in its shell. Consequently, in the formation of NaCl, Na is going to give electrons and Cl is going to get them. As a result of this we can write Na+Cl-. Here Na+ means +1 charge because the sodium is giving the electron, while Cl- means -1 charge because the chlorine is getting it. The bond is ionic.
But otherwise, if the bond is covalent, we'd better focus on partial positive or negative charges. In the formation of the molecule of water, the oxygen **is gaining** 2 electrons from the 2 hydrogens, which **are losing** them, the Hs being more electropositive and the O being more electronegative. Consequently the oxidation number of hydrogen in H2O is +1, while the oxygen's is -1. As a result of this we can say that in a molecule of water the hydrogen **are oxidized** by the oxygen: the electrons **are taken away** from them, so that they **have** a positive charge.

Now, let's study the following combustion:

\[ \text{CH}_4 + 2\text{O}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O} \]

Here a molecule of methane **is reacting** with two molecules of oxygen in order to produce a molecule of carbon dioxide plus 2 molecules of water plus some heat (being esothermic, the reaction **produces** more heat than you put into it). In CH4 an atom of carbon **is bounded** with 4 hydrogens. While reacting, being more electronegative, the carbon **is taking** 4 electrons from the hydrogens, so its charge **is going** down by four. As a result its oxidation number is -4 , while the hydrogen's is +1. Thus, we can write C-4H+1. In CO2, the carbon's oxidation state is +4, which **means** that it **is giving up** 4 electrons, and really it only **has** 2 electrons to give up, for it **has** 4 electrons in its valence shell. So, what **is getting** oxidized and what **is getting** reduced? Let's write down the first half reactions:

\[ \text{C} - 4\rightarrow \text{C} + 4 + 8\text{e}^- \]

Here carbon **is going** from an oxidation number of -4 on the left side of this equation, to an oxidation number of +4 on the right side: 8 electrons **are being taken away** from carbon, so it **is being oxidized**. As for the second half reaction

\[ 4\text{O} + 8\text{e}^- \rightarrow 4\text{O}_2 \]

we **have** 4 oxygens with a zero oxidation state (being in the elemental form) **turning into** 4 oxygens with a -2 oxidation state, so each of these oxygens **are taking** 4 e-, the two of them, so that there are 8e-. The oxidation state, that is the hypothetical charge, **is going down**, or it **is being reduced** by carbon, as well as the carbon above **is being oxidized** by oxygen. Finally, what's the oxidizing agent, what is the thing that is oxidizing? Of course the oxygen is the oxidizing agent, while carbon is the reducing agent.

Redox can also be reviewed from a biological point-of-view. Biologists **usually say** oxidation **deals with** losing hydrogen atoms, while reduction **deals with** gaining hydrogen atoms, though the essential meaning **stays** the same.

The reactions within cells which result in the ATP (adenosine triphosphate) synthase using energy stored in glucose **are referred to** as cellular respiration. It **requires** oxygen as the final electron acceptor. The equation for aerobic respiration is

\[ \text{C}_6\text{H}_12\text{O}_6 + 6\text{O}_2 \rightarrow 6\text{CO}_2 + 6\text{H}_2\text{O} + \text{energy} \]

Here we **are combining** glucose with molecular oxygen so that cellular respiration **is being performed**. We **end up** with 6 carbon dioxides and six molecules of water, while the energy produced is **made up** of some heat and about 38 ATPs. Glucose **is completely broken down** to CO2 + H2O though, during fermentation, it **is only partially broken down**.
Let’s take a look at the half reactions:

\[ \text{H}_2 \rightarrow 6 \text{H}_2 \rightarrow \text{hydrogen yielding to 6 hydrogens} \] says the hydrogen preserves a +1 oxidation number (o.n.) on both sides of the equation so that nothing is happening with respect to oxidation and reduction, while \( \text{C}_6 \rightarrow 6\text{C} + 24\text{e}^\text{-} \) shows the number of electrons lost by carbon in cellular respiration, the carbon being oxidized by the oxygen. Finally, \( 6\text{O}_2 + 24\text{e}^\text{-} \rightarrow 6\text{O}_2 + 6\text{O} \) emphasizes the fact that these 24 electrons are the same electrons carbon is losing, so that oxygen, which is gaining electrons, is being reduced by carbon.

Hence, where does the energy come from? The energy is produced because the electrons are going from a higher energy state, or level, to a lower one (we know that lower orbitals are more stable):

\[
\begin{align*}
\text{C}_6\text{H}_{12}\text{O}_6 + 6\text{O}_2 & \rightarrow 6\text{CO}_2 + 6\text{H}_2\text{O} \\
\uparrow & \downarrow \swarrow \checkmark \\
\text{Oxidized} & \quad \text{reduced} & \text{e- are going} \quad \text{to these oxygens}
\end{align*}
\]

that is to say carbon is losing hydrogens, while oxygen is gaining hydrogens,

### EXERCISES

Text building: use the following redox reactions to build a coherent text with appropriate verbs in the present tense (active and passive forms) and consequential connectives such as so, so that, consequently, as a result of, thus, hence:

1) \( \text{Zn} + \text{CuSO}_4 \rightarrow \text{ZnSO}_4 + \text{Cu} \)
2) \( \text{Fe} + 2\text{HCl} \rightarrow \text{FeCl}_2 + \text{H}_2 \)
3) \( \text{H}_3\text{PO}_3 + \text{MnO}_4 \rightarrow \text{HPO}_4 + \text{Mn} \)
4) \( 2\text{H}_2 + \text{O}_2 \rightarrow 2\text{H}_2\text{O} \)

### QUANTIFIERS

Let’s read the following text to be found at http://press.princeton.edu/chapters/gowers/gowers_I_2.pdf:

Let’s consider the statements
(1) Nothing is better than lifelong happiness.
(2) But a cheese sandwich is better than nothing.
(3) Therefore, a cheese sandwich is better than life-long happiness.
We may reflect upon the word “nothing”, which is used in two different ways.
The first sentence means, “There is no single thing that is better than lifelong happiness,” whereas the
second means, “It is better to have a cheese sandwich than to have nothing at all.” In other words, in the second sentence, “nothing” stands for what one might call the null option, the option of having nothing, whereas in the first it doesn’t (to have nothing is not better than to have lifelong happiness). Words like “all”, “some”, “any”, “every”, “nothing” are called quantifiers, and in the English language they are highly prone to this kind of ambiguity. Mathematicians therefore make do with just two quantifiers, and the rules for their use are much stricter. They tend to come at the beginning of sentences, and can be read as “for all” (or “for every”) and “there exists” (or “for some”). A rewriting of sentence (1) that renders it unambiguous (and much less like a real English sentence) is (1)’ For all x, lifelong happiness is better than x. The second sentence cannot be rewritten in these terms because the word “nothing” is not playing the role of a quantifier. (Its nearest mathematical equivalent is something like the empty set, that is, the set with no elements.). Armed with “for all” and “there exists”, we can be clear about the difference between the beginnings of the following sentences.

(4) Everybody likes at least one drink, and that drink is water.
(5) Everybody likes at least one drink; I myself go for red wine.

The first sentence makes the point (not necessarily correctly) that there is one drink that everybody likes, whereas the second claims merely that we all have something we like to drink, even if that something varies from person to person. The precise formulations that capture the difference are as follows.

(4)’ There exists a drink D such that, for every person P, P likes D.
(5)’ For every person P there exists a drink D such that P likes D.

This illustrates an important general principle: if you take a sentence that begins “for every x there exists y such that ...” and interchange the two parts so that it now begins “there exists y such that, for every x, ...”, then you obtain a much stronger statement, since y is no longer allowed to depend on x. If the second statement is still true -that is, if you really can choose a y that works independently of x - then the first statement is said to hold uniformly.

Finally, the symbols $\forall$ and $\exists$ are often used for “for all” and “there exists” and are, like all quantifiers, always associated with a set (one says that it quantifies over that set), such as, for instance, $\forall a, b \in \mathbb{N}$.

EXERCISE. Write a text answering the following questions: “What do we use quantifiers for?” , “How do they work in the context?”

QUANTIFYING DIFFUSION, OSMOSIS AND THE IDEAL GAS

Say I have a container with a bunch of water molecules bumping to each other. Inside I have also sugar molecules, though I have many more water molecules. We call the thing there’s more of the solvent (in this case water). Whatever there is less of it’s the solute. So we say that the sugar has been dissolved into the water. The combination of the two we call it the solution. The solvent is the thing doing the dissolving and the thing that is dissolved is the sugar, that’s the solute. We want to focus on diffusion. Let’s consider another container with some molecules of gaseous oxygen randomly bumping to each other with a certain amount of kinetic energy. What is it going to happen in this type of container? After a certain time the system reaches its equilibrium. The particles, being initially somewhere closed to each other are going to get relatively spread out. This is diffusion: essentially the spreading of
molecules from high concentration to low concentration areas. The key question are: “How many particles do I have per unit space?” , “How much solvent is there?” , “How much solute?”

Say I have more than one container, let’s say two, with some water inside and with a hole in the middle; I have a bunch of water molecules on either sides. Assuming that both sides have the same level of water, the particles going from the right to the left equal those going the opposite direction with the same pressure. Let’s dissolve some solute in it on the left end side so that they are small enough to fit through the hole, if there’s any, of course. Over time some of the sugar molecules will go to the other side so that the concentration might be equal: this is an example of diffusion of the solute. We have a higher concentration = lower concentration system –that is, a shift from a hypertonic solution to a hypotonic one.

What happens if I have a tunnel where the solute is too big to travel, but water is small enough to travel? Let’s say we have an outside environment with a bunch of water around a semipermeable membrane, so that water can go in and out the membrane (permeable to water), but the solute can’t go out of it, its molecules being too big. Zooming the membrane we realize sugar can’t get through the holes, only water can get back and forth. Which side of the membrane has a higher or lower concentration of solute? The inside of the membrane is hypertonic, the outside has a lower concentration, so it’s hypotonic. If there were no sugar molecules water would have had equal likelihood (probability) of going in both directions, but being there, they prevent some molecules of water from passing the hole in one direction, but not in the other. So we have inward flowing of water to try to equilibrate the concentration. So we have flowing of the solvent from a hypotonic situation to a hypertonic solution, so that the membrane stretches out. Diffusing through a semipermeable membrane is called osmosis. Over time the pure solvent dilutes the solution, whose level jumps up, while the solvent’s gets down. If we consider a U-shaped tube this goes on until the pressure exerted on the membrane’s side touching the solution (hydrostatic pressure) balances the osmotic pressure—that is, the pressure exerted by the solvent molecules approaching the solution. So the pressure exerted by the solvent is balanced by an equal and opposite pressure that prevents the solvent from passing. We can thus quantify the osmotic pressure as the pressure we have to apply on a solution to prevent the solvent from passing into it. To quantify all this we can write

\[ \pi V = nRT \]

where \( \pi \) is the osmotic pressure, \( V \) is the volume, \( R = 0.0821 \text{ L atm/mol K} \) \( = 8.3145 \text{ m}^3 \text{ Pa/mol K} \) is the universal gas constant and \( T \) is the temperature an \( n \) is the number of moles, that is the amount of substance of a system that contains as many particles (atoms, molecules, ions, electrons, nuclei, etc.) as there are atoms in 12 grams of 12C (which means that there are 1 mol of 12C atoms in 12 grams of carbon, so that there are 6.02 x 10^23 (Avogadro’s number) atoms in 12 g of C. If the concentration \( c = n/V \) then

\[ \pi = cRT \]

EXAMPLE: human blood’s plasma has an osmotic pressure of 7.65 atm at a temperature \( T=37 ^\circ \text{C} \). How many grams of glucose must be dissolved in water so that the solution be isotonic with the blood’s plasma (glucose molar mass = 180.16 g/mol) supposing the final volume being constant?

\[ P=nRT/V = wRT/MV \Rightarrow w = PMV/RT = 7.65 \text{ atm} x 180,16 \text{ (g/mol)} / 0.0821(\text{L atm/mol K}) x 310K = 54,15 \text{ g/l} \]

The injected solution must be isotonic with the plasma because if it were more diluted, the water would flow inside the cells where there’s a greater concentration of solutes, swelling them until breaking. If the injected solution were more concentrated than the cellular liquid then the opposite would happen: the cells would lose water becoming wrinkled.
Say I have a container with a lot of gas molecules bumping to each other and onto the container, all with their velocity vectors. Let’s assume that at each bump that occurs there’s no loss of energy, no loss of momentum: it’s an ideal gas. We can think of pressure as something pushing on an area, so \( p = \frac{F}{S} \). If we consider a side of the container, every time the molecules bump they have a change in momentum, so the force exerted on the wall will be given by the change in momentum over the change in time: \( F = \frac{\Delta p}{\Delta t} \). Roughly speaking, the product of the pressure exerted at any point of the container and its volume is proportional to a constant –that is, the kinetic energy of the molecules bumping around, so \( pV = K \). If I were able to decrease the volume of the container by squeezing it, for instance, the molecules would hit the sides more often, there would be more changes in momentum, each particle would exert more force, so that the pressure would definitely be higher. So if volume goes down, then pressure goes up. If I make the volume bigger, then it will take longer for the molecules to bump against larger surfaces. So if the volume goes down, then pressure goes up.

EXAMPLE. Let’s say I have a box with an initial volume of 50 cubic meters (or meters cubed), so \( V_1 = 50 \text{ m}^3 \), and an initial pressure of 500 Pascal, so \( P_1 = 500 \text{ Pa} = \frac{N}{\text{m}^2} \). Then I squeeze the container down to 20 m\(^3\). What’s the new pressure? There’s no work on the system, no exchange of energy from outside of the system, just the squeezing, so \( P_1 V_1 = P_2 V_2 \Rightarrow 500 \times 50 = 20 \times P_2 \Rightarrow P_2 = 1250 \text{ Pa} \).

As for temperature we can essentially say that it is, through a constant \( K \), the ratio of the average kinetic energy of the system to the number of molecule, \( T = k \frac{E}{N} \), so that \( E = NT/k \). We’re essentially saying that \( pV = (1/k) NT \). That’s to say that \( P_1 V_1 / T_1 = P_2 V_2 / T_2 \). If the temperature inside my container goes up, then the particles will bump more, so that also the pressure will go up, assuming the volume stays flat. If \( T \) goes up and the pressure stays flat, the only way to make the molecules bump across is to increase the volume while increasing the temperature. Roughly speaking, we can consider the moles as being the ratio of the mass in gram to the molar mass and the universal gas constant \( R=8.31 \text{ J/mol L} \), thus rewriting the equations above as Boyle’s law \( pV = nRT \)

**EXERCISES:** write about the following situation building up a text with appropriate quantifiers.

1) I’ve got some gas in a balloon at a \( P_1 = 3 \text{ atm} \), \( V_1 = 9 \text{ liters} \) and \( T=\text{constant} \). What will pressure become if \( V_1 \) goes from 9 liters to \( V_2 = 3 \text{ liters} \)?

2) If I have an ideal gas at a STP, how many moles do I have in 1 liter? –that is, how many liters will one mole take up?

3) I have a container with \( p=12 \text{ atm} \), \( V=300 \text{ ml} \), \( T=10^{\circ}\text{C} \). How much O2 in grams is there?

4) I have 98 ml of an unknown substance, its mass being 0.081 g at STP. What’s the missing substance?

4) Say I have a balloon of He, \( V=1\text{m}^3 \), \( P=5\text{Pa} \), \( T=20^{\circ}\text{C} \). Calculate the number of moles.
In order to deal with past tenses we may use the following scheme and apply it according to our lab’s system moving across a timeline.

**Used to:** we use it when referring to a system we used in the past but we are not using any longer. The general rule is:
- **Positive:** S + used to + base form (1st column of the verbal paradigms)
- **Negative:** S + didn’t use to + base form (1st column of the verbal paradigms)
- **Interrogative:** Did + S + used to + base form

**Past simple:** we use it when our system, as any dependent variable approaches infinity from any direction, or any finite value we realized one second ago to be useless, is definitely dismissed. Adverbs: last + any time expression, ago, yesterday, in a past year, etc.
- **The scheme is:**
  - **Positive:** S + past simple (2nd column of the verbal paradigms)
  - **Negative:** S + didn’t + base form (1st column of the verbal paradigms)
  - **Interrogative:** Did + S + base form

**Past continuous:** we use it when referring to a system that was working at the same time of any system we dealt with or used to deal with in the past:
- **Positive:** S + was/were + -ing form of the following verb
- **Negative:** S + wasn’t/weren’t + -ing form of the following verb
- **Interrogative:** was/were + s + -ing form of the following verb

**Present perfect simple:** we use it any time we need to connect our previous system to the time of speaking –that is, a system we have already done or referred to. Adverbs: just, already, ever/never, recently, yet, so far, and the like.
- **Positive:** S + have/has + past participle (3rd column of the verbal paradigms)
- **Negative:** S + haven’t/hasn’t + past participle
- **Interrogative:** Have/has + S + past participle

**Present perfect continuous:** we use it any time we need to emphasize how long our system has been working and the fact that it is still working. To specify the duration we use “for”; we use “since” to enhance the beginning of the duration.
- **Positive:** S + have/has been + -ing form of the following verb
- **Negative:** S + haven’t/hasn’t been + -ing form of the following verb
- **Interrogative:** Have/has + S + been + -ing form of the following verb

**Past perfect simple:** we use it any time we refer to a system we had used before any other system we used in the past:
- **Positive:** S + had + past participle (3rd column of the verbal paradigms)
- **Negative:** S + hadn’t + past participle
- **Interrogative:** Had + S + past participle

**Past perfect continuous:** we use it any time we need to emphasize how long our system had been working up to the past system we refer to. To specify the duration again we use for and since.
- **Positive:** S + had been + -ing form of the following verb
- **Negative:** S + hadn’t been + -ing form of the following verb
- **Interrogative:** Had + S + been + -ing form of the following verb
How long will it take an A439 airbus to take off? How much distance will it cover?

I was concentrating on my system when I realized I had been comparing two measurements for quite a while, though they had to be of course identical! I used to take precise notes about that but, for similar systems, I had already found out it was a waste of time. I’ve been thinking about that since I entered the lab, but now I want to be very clear!

We know that yesterday the take-off velocity of an A439 airbus of the American Airlines had a magnitude of 280 Km/h in the direction of going down the runways. From the moment when it left to the actual take-off it had a constant acceleration of 1.0 (m/s)/s or 1.0 m/s^2 (that is, 1.0 meters per second per second: after every second it could go 1 meter per second faster than it was going at the beginning of that second). How long did the take-off last? We can convert kilometers per hour into meters per second:

280 km/hour * 1 hour/3600 sec * 1000 m / 1km = 78 m/sec

which is pretty fast: we have just realized that for every second that went by the airbus has travelled 78 meters, roughly three fourths the length of a football field! So how long would it take to go 78 meters per second? Just 78 seconds. And it makes sense, for acceleration = change in velocity / change in time \( \Delta t = \Delta v/a = 78 m/s / 1.0 m/s^2 = 78 \text{ sec} \). Consequently, yesterday the airbus took about seventy-eight seconds to take off.

Today we are being asked this question: How long of a runway does the same airbus need? Given these numbers, what is the minimum length of the runway? Actually, we want to figure out the displacement (a vector point that represents the difference in the position of two points), or how far does this plane travel as it accelerates 1 m/s^2 to 280 km/h, how much length does this thing cover – that is, we are going to calculate the airbus A380 take-off distance. If we assume the acceleration to be constant, then we’ll end up with something called the average velocity, which is \( V_a = (V_f + V_i) / 2 = 78 + 0 / 2 = 39 \text{ m/s} \). We can now calculate the displacement: \( s = V_a \Delta t = 39 \text{ m/s} * 78 \text{ s} = 3042 \text{ m} \). So we need over three kilometers for one of these stuff to take off (1.8/1.9 miles). If we graph this in a v(t) system of coordinate axis we’ll find out that the area under the curve given by the constant acceleration up to the 78 s over the t-axes is the distance travelled, and if we take the average velocity for the same amount of time we will get the exact same area under the curve, or we will take the exact same
distance. Of course the distance it is covering today is the exact same distance it covered yesterday as well as yesterday’s take-off velocity, given those numbers, would be the same today! The relative Physics does not change in a couple of days! It does the way we talk about it—that is, we represent it over a tense-line (vs. a time-line)!

Generally speaking, we may wonder why distance is the area under the velocity-time line. Well, let’s say I have something moving to the right with a constant velocity (as the seconds stick away it does not change) of 5 m/s and let’s plot it against time. How far does this thing travel after 5 seconds? We know that \( v = \frac{\Delta s}{\Delta t} \Rightarrow v \Delta t = \Delta s \Rightarrow 5\text{m/s} \times 5\text{s} = 25\text{m} \). This is exactly the area under the rectangle we obtain by plotting the graph. The area is going to be the distance travelled because displacement is the product of velocity to the change in time. Let’s assume \( a = \frac{\Delta v}{\Delta t} = 1\text{m/s}^2 \) and the magnitude of initial velocity \( ||v|| = 0 \). After 1 sec I’m going 1 meter per second faster, after 2 seconds I’m going another meter per second faster than that, again when I go forth in time, I’ll go one second faster (and so up) than that: if we remember the definition of the slope we can draw the line corresponding to the acceleration, which is the slope of that line. So if we accelerate that amount for 5 seconds, how far have we travelled? For every second I can split up a rectangle: the smaller the rectangle, the closer we are going to get to the area under the curve, which is the distance travelled, which is luckily going to be the area of a triangle, \( A = \frac{1}{2}bh/2 \). So the magnitude of the displacement (distance) \( ||s|| = \frac{1}{2}(5\text{s} \times 5\text{m/s}) = 12.5\text{m} \).

EXERCISE. Can you name the tenses in bold with respect to the scheme above? Which is thei function in the context?

EXERCISE. Calculate the acceleration of aircraft carrier takeoff with a null initial velocity, a final velocity of 260 km/h and a displacement of 80 m and write a coherent past/present text. Focus also on the duration.

VERBS, PARTICLES AND PHRASAL VERBS

So far we have realized that the uses of prepositions is peculiar to English language; verbs too are almost always followed by prepositions in order to better fit their meaning with the system of coordinate axis we continuously reproduce when speaking. Otherwise, we use a combination of verb and one or more adverbs or prepositions, as catch on, take off, or put up with, functioning as a single semantic unit and often having an idiomatic meaning not predictable from
the meanings of the individual parts: these are the so-called phrasal verbs. Let’s read the following text about

WATER’S SELF-IONIZATION

Say we have the following reactions:

\[
2\text{H}_2\text{O} \, (\text{aq}) \rightleftharpoons \text{H}_3\text{O}^+ \, (\text{aq}) + \text{OH}^- \, (\text{aq}) \quad (1)
\]

\[
\text{H}_2\text{O} \, (\text{aq}) \rightleftharpoons \text{H}^+ \, (\text{aq}) + \text{OH}^- \, (\text{aq}) \quad (2)
\]

They both represent the autoionization of water. Just to give a narrative around what could happen, we can say that reaction (1) shows up two reacting molecules of water yielding to an hydronium ion and a hydroxide: water is a neutral molecule and as soon as the oxygen’s δ- of one molecule gets attached to the hydrogen’s δ+ of the other one we are left with one less proton than electrons in the products. The hydrogen’s nucleus itself, on the left side of the equation, which is a proton, gets bumped off or scraped off and ends up on the hydronium molecule on the left side, while the hydroxide has a negative charge because it has lost a proton; the hydronium has the same number of electrons, but now has got an extra proton, so it has a positive charge. This process where water can kind of spontaneously bring about such a performance is called autoionization because you just have water by itself, and just buy some random circumstances of some molecules bumping into each other just right, some subset of the water will ionize like this, where one part will lose a proton and the other part will gain one. Of course this is an equilibrium reaction, so now the two molecules might go bumping into two other and become water again; maybe they’ll bump into each other again and become water again, so goes back and forth, so that there’s some equilibrium concentration of this, the proper equilibrium reaction being in the form (1). Anyway, equations (1) and (2) are essentially equivalent, although the first describes what exactly happens. The second one is creating a sort of picture: we have 1 water molecule and there’s some small probability that one of its hydrogens just pop off, and we’re just left with a hydrogen and a hydroxide, which is just an oxygen and a hydrogen atom together, but the reality is that the H’s do not exist in water on their own. Whenever they are in water, in an aqueous solution, they essentially get a ride with another water molecule, and that’s what happens with the hydronium ion. But otherwise, since people care about hydrogen cations—that is, hydrogen protons in a solution, we more often consider the (2). What is the equilibrium of this reaction? It turns out that in just regular water, or pure water, at 25°C, which is roughly room temperature we have \([\text{H}_3\text{O}^+] = [\text{OH}^-] = 10^{-7}\) M (molar): for every 1 liter of water we have 10 to the minus 7 moles of hydronium cations, or hydrogen’s (that is, 10 to the minus 7 times Avogadro’s number, which is pretty a good number, for if you do the maths you end up with 60 times 10 to the 16th power molecules!). the concentration of hydroxide is also \(10^{-7}\) M. if we know that we can now figure out the equilibrium constant for the autoionization of water in (2), which we’ll call for the Kw sub w for water: \(K_w = [\text{H}^+] [\text{OH}^-] = 10^{-14}\). In a normal situation we would divide this by the concentration of the reactants, but in this case the reactants are what the solvent is: actually the probability that what is on the left side of the equation turns into what is on the right side is maximum, it’s equal to 1, so we should divide by 1—that is, not dividing at all! People chemistry don’t like talking in this way, they like taking the negative logarithm (which is base 10) on both side of the equation (and if the log is intimidating to you remember it’s just an exponent!), so \(-\log_{10} K_w = -\log_{10} 10^{-14} = 14\): this idea right here is called a pKw, any p in chemistry standing for \(-\log_{10}\), the p standing for power. The same convention is used in the hydrogen concentration. Since \([\text{H}^+] = 10^{-7}\) M,
we end up with a pH = 7 = -log [H⁺]. Of course if we add H⁺ up to get [H⁺]=10⁻³, the solution thus becoming more acid, the new pH will equal 3. We can do the same exercise for the pOH, which turns out to be equal to 7, because water has the same number of hydrogens and hydroxides, for it is dissociating into the two of them.

**EXERCISE:** Focus on the verbs in bold. Which are just verbs followed by prepositions and which are phrasal verbs? What’s the difference, if any? Explain them in the context and find out any synonyms.

**EXERCISE:** Calculate the pH of 10⁻⁸ M of a HCl solution and build up an appropriate phrasal-verb text.

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**SENTENCE STRUCTURE: ADJECTIVES’ ORDER**

The use of adjectives is essential when trying to describe a noun or pronoun. Adjectives are always singular in English even though the noun is plural and even though they are an infinite number. But otherwise, if more than one, they should follow a certain order, which is roughly the following
ORDERING ADJECTIVES: A MATTER OF GENETICS AND HEREDITY?

We have the general sense that offspring is the product of parents’ traits, some of which being dominant. In most cases, for instance, when brown-eyed men (that is, men who have got brown eyes) marry blue-eyed women (that is, women who have got blue eyes) their baby, or babies, will generally be brown-eyed. Already in the middle of the XIX century, before we knew what DNA (Deoxyribonucleic acid) was discovered, Gregor Mendel introduced the conceptions of genes and alleles, the latter being a specific version of a gene. Today, in Genetics we know that a chromosome is a singled piece of coiled DNA containing many genes. To oversimplify things we can say that particular genes, such as eye color or tooth size, are located at a same certain point of a chromosome from both the parents: they are called homologous chromosomes. If I call one point b (that stands for blue eyes) and the other B (that stands for brown eyes), then I can write my genotype as Bb. Each of the way this eye-color gene is expressed is an allele, so that the b and the B in Bb are two different alleles, or versions of the same gene. When I have two different versions of this, one from my mom and the other from my dad, I’m called a heterozygous genotype (or a hybrid, though this is a bit old-fashioned term), and the genotype is the exact version of the allele I have. If I were a lowercase b / lowercase b, or a upper case B / upper case B, then I would have two identical alleles, both of my parents giving me the same version of the gene. In this case I am a homozygous genotype, or I am a homozygote for this trait. Assuming the idea that Brown are dominant traits and blue are recessive traits. This means that if I were to inherit the Bb genotype, because the B allele is dominant, all you are going to see for the person with this genotype is brown eyes. In this way, a genotype is the actual version of the gene you have and a phenotype is what’s expressed, or what you see. So many different genotypes are all coded for the same phenotype, or phenotype-coded genotypes: Bb, bB, BB = brown eyes; bb = blue eyes. Considering, for example, my parents as being both heterozygous, we can build a mono-hybrid cross –that is, a grid known as punnett square, which is useful for each kind of crosses between two reproducing organisms:

\[
\begin{array}{cc}
B & b \\
B & BB & Bb \\
b & bB & bb
\end{array}
\]

where we place the parents’ chromosomes and build up the possible combinations. Of course, if I get the brown-eye allele from my dad and the brown-eye allele from my mom, I will be a BB genotype, etc. What’s the probability I’m going to have brown eyes? Each of the 4 scenarios are equally likely, so I put a 4 at the denominator; the scenarios ending up with B phenotypes are 3, so I put it at the numerator. Thus my \( P(B) = \frac{3}{4} = 75\% \), while my \( P(A) = 1 - \frac{3}{4} = \frac{1}{4} \), which is the probability the two parents produce an offspring with blue eyes. If one of my parents is homozygous BB, for instance, and the other heterozygous Bb, then \( P(bb) = 0 \), while the probability to get a homozygous dominant is of course \( \frac{2}{4} = 50\% \) shot.

There is also a case of an incomplete dominance, when no trait is dominant: you might have mixing or blending of the traits when you look at them. Let’s consider Red flowers and White flowers so that

\[
\begin{array}{cc}
R & W
\end{array}
\]
where RW reads ‘pink’, a blending of the two. Of course P(pink) = \( \frac{1}{2} \). We can have also a situation in which we have multiple different kind of alleles, for example in blood types, where there are 3 potential alleles: A, B, O. We are going to have a combination between co-dominant and recessive genes. An AB blood-type parent means that on one of their homologous chromosome they have the A allele and on the other one they have the B allele: the phenotype is the genotype, they are co-dominant, they do not necessarily blend, but they both express themselves. The other parent is fully an A blood type: the phenotype is an A blood type, but the genotype has one A allele and one O allele:

\[
\begin{align*}
A & \quad AA & \quad AB \\
O & \quad AO & \quad BO
\end{align*}
\]

O is recessive, while A and B are co-dominant –that is, an AO combination brings about an A phenotype. Thus P(A) = 50%: 2 out of 4 combinations show us an A blood type.

A punnett square can be useful also when we talk about more than one trait. Let’s consider t=small teeth and T=big teeth. So one can be heterozygous for brown eyes and homozygous for big teeth, the combination being BbTT and the phenotype being a big-toothed brown-eyed person. If we assume that these genes are located on different chromosomes (two homologous pairs), then they will assort independently. That’s what we call independent assortment. After meiosis occurs to produce the gametes the offspring might copy a chromosome for uppercase color and may get one for big tooth-size (upper case, too), or high color and small tooth-size, that is, another version of the allele, and so on. If we add one more trait, say hair color, the genes being on homologous pairs 1, assuming the blue-eyed/blond-haired trait to be on one same chromosome of the pairs, they are always going to ‘travel’ together, they do not assort independently any longer: they are called linked traits. Let’s say both parents are di-hybrids for the genes BbTt, or BbTt-gene di-hybrids, what are going to be the combinations for their children? For each of these traits the mom, say, can only contribute one of the alleles, for instance BT or Bt or bT or bt; the same for dad, of course. Thus

\[
\begin{align*}
BT & \quad Bt & \quad bT & \quad bt \\
BT & \quad BBTT & \quad BBtT & \quad BbTT & \quad BbTt \\
Bt & \quad BBtT & \quad BBtt & \quad BbTT & \quad Bbtt \\
bT & \quad BbTT & \quad BbTt & \quad bbTT & \quad bbTt \\
b t & \quad BbTt & \quad Bbtt & \quad bbTt & \quad bbtt
\end{align*}
\]

There are 16 possible combinations. What sort of phenotypes might be expressed from this di-hybrid cross? How many of these are going to exhibit brown eyes and big teeth? A big-toothed brown-eyed kid has a probability of 9/16; a brown-eyed little-toothed has a 3/16 and a blue-eyed big-toothed again a 3/16, while a recessive-on-both-trait kid has just 1/16 probability to inherit those same traits.

On another level, if we assume B and b to be the only possible combinations and also that there is no natural selection, no mutation and a large population in order to have a stable allele frequency with respect to the eye color gene. In a population of simply a Bb and a bb, for instance, the B frequency is 75%, while the B’s is of course 25%, whereas the phenotype frequency is 50%. With the above assumptions, anyway, the allele frequency is going to be constant and I’m in a Hardy-Weinberg
equilibrium: the allele frequency (f) is not changing. In this way we can observe and deduce a lot of things, such as the genotypes in the population, or the frequencies of different phenotypes or whatever else. So for example if p = frequency of b and q = frequency of B, then p + q = 1 (100%). To get the frequency of the genotypes we can square both sides of this equation, so that p^2 + 2pq + q^2 = 1. Now p is the probability that I get two of the lowercase blue alleles – that is, I end up with a bb combination (p being the probability that I get a blue from each of my parents, so that’s a p squared). The same for q squared. The entire 2pq term is instead the frequency that I have a heterozygous genotype, for there are two ways that I can be a heterozygous: Bb and bB, that is pq and qp that, added together lead to 2pq, which is essentially the frequency of the hybrids in the population. So for example if in a population of 1 million we observe a 9% of b frequency – that is a bb, because being recessive, we need 2 copies. So p^2 = 9/100. So the frequency of the blue allele in the population is p = sqrt(0.09) = 0.3. So if you counted all of the alleles in the population, you would actually find that 30% of them are the lowercase blue. We had a smaller percentage because you need two of them: you have a 30% chance of getting it from your mom and a 30% from your dad. On the other side, the frequency of a brown-eyed allele is of course 75/100 = 75%, given that (p + q)^2 = 1. If 30% of all the chromosomes or of all of the alleles in the population are blue, the other 70% are going to have to be brown. Reasoning like that, of course, the brown-eyed population must be 91%. What percentage of the population are going to be homozygous for brown eyes? They need to have an uppercase B from both parents, so BB = q^2 = (0.7)^2 = 0.49 = 49%, which is the percentage of the population that is homozygous dominant. The remainder are hybrids, that is 42%. Going back to Hardy-Weinberg equation we have p + q = 1 \approx 30% + 70% = 1, consistent with p^2 + 2pq + q^2 = 1 \approx 9% + 42% + 49% = 1. As for the phenotypes, of course, they are 9% vs 91%.

EXERCISES

Write a coherent text using if clauses 1, 2, 3 to describe the grids (1), (2), (3).

Use the punnett square (4) as specimen introduction to create a similar grid with other traits and build up a text with appropriate adjectives. Use also if clauses, relative clauses, logical connectives and ‘when’, ‘as long as’, ‘unless’, ‘as soon as’.

In a large population we have 12% of any recessive allele frequency. Calculate the percentage of the genotypes and the phenotypes and build up the corresponding text.

Find out any case of incomplete dominance and independent assortment, build up the corresponding crosses and fit the entire system into a coherent and cohesive text.

We want to briefly figure out how a scientific topic may be treated in order to sketch a coherent text about it. Such a text should contain a pretty straightforward introduction, the ‘body’ of the text including graphs, numerals, equations, etc., and, of course a conclusion.

Let’s choose our topic as
ENTHALPY

We want to figure out and quantify the conception of enthalpy of formation of a substance.

To this extent, let’s consider a P(V) diagram, where P is the pressure and V is the volume of the system:

We know that the area under the curve in the clockwise direction is the net work done by the system. Here $\Delta U = 0 = \text{change in internal energy of the system.}$

Anyway $\Delta U = Q - W = 0$

Where Q is the heat applied to the system and W is the work done by the system.

We define enthalpy as $H = PV$. If we consider a finite change in H we can write

$$\Delta H = \Delta U + \Delta(PV) = Q - W + \Delta(PV)$$

Here $\Delta H$ is a state variable because it is the sum of other state variables

Considering a system with a piston we can write

$$\Delta H = Q - P \Delta V + \Delta PV$$

We realize the change in enthalpy will equal Q if the last two terms cancel out. Under what condition? IF and only if the pressure is constant, then I can factor it out.

We get

$$\Delta H = Q - P \Delta V + P \Delta V = Qp$$

That is, heat at constant pressure.
We realize we kind of squeezed out the previous diagram, for we’ve made of the forth path and the return one the same exact path—that is, a horizontal line from A to B. As a consequence, no net work has been added in going from A to B.

Most chemical reactions are at constant pressure (1 atm). In this case we can define enthalpy as the heat content when pressure is constant.

Example. Consider the following reaction:

\[ C(s) + 2H_2 (g) \rightarrow CH_4 + 74 \text{ KJ of heat released} \]

How much heat is being added to the system? We realize \( H_{\text{initial}} = \text{reactants} > H_{\text{final}} = \text{products} \) and that

\( Q_p = -74 \text{ KJ} \) (that is, the change in enthalpy). This heat has been added to the system, so what’s the change in enthalpy of the reactants’ system relative to the products’ system?

We can write

\[ H_f - H_i = \Delta H = -74 \text{ KJ} \]

That is, \( H_f \) is lower than \( H_i \) by 74 KJ, so we realize \( H_f \) is at a lower level of energy, that is, it’s more stable and that the reaction is esothermic, that is, the heat is released by the system.

Most chemical reactions are at constant pressure (1 atm). Thus we can define enthalpy of formation of a substance as the heat content when pressure is constant.

**EXERCISE.** Calculate the variation of the standard enthalpy of formation for the process, at constant pressure, \( C(\text{diamond}) \rightarrow C(\text{graphite}) \) knowing that

\[ C_{\text{graph}} + O_2 (g) \rightarrow CO_2 \quad \Delta H^\circ_f = -393.5 \text{ KJ/mole} \]

\[ C_{\text{diam}} + O_2 (g) \rightarrow Co_2 (g) \quad \Delta H^\circ_f = -395.4 \text{ KJ/mole} \]

You must write a coherent English text by fitting the numerals and the chemicals equations within it.
LOOPS AND CLAUSES

In computer programming, a **loop** is a sequence of instructions that is continually repeated until a certain condition is reached. Typically, a certain process is done, such as getting an item of data and changing it, and then some condition is checked such as whether a counter has reached a prescribed number. If it hasn't, the next instruction in the sequence is an instruction to return to the first instruction in the sequence and repeat the sequence. If the condition has been reached, the next instruction "falls through" to the next sequential instruction or branches outside the loop. A loop is a fundamental programming idea that is commonly used in writing programs: on a linguistic level, it is a construct, or a clause. Very often when you write code, for instance, you want to perform different actions for different decisions. You can use conditional statements in your code to do this. In most codes, such as Java script, Fortran, Scilab, Matlab, C, IDL we use a
bunch of statements, more or less nested, that allow us to construct our program, or text. We use an \textbf{if} statement to execute some code only if a specified condition is true or an \textbf{if...then}...\textbf{else} statement to execute some code if the condition is true and another code if the condition is false. Again, an \textbf{if}....\textbf{elseif} loop executes a set of instructions if some condition is true or if some other conditions are true. A \textbf{while} loop, on the other hand, is a set of instructions that is being repeated \textbf{until} a particular condition is satisfied. In order to execute a set of instructions a determined number of times, then, we use a \textbf{do} loop -that is, an iterative loop - \textbf{for} the number of times you equaled the counter to. We can also use a where clause, especially in database programming such as SQL, that specifies which data values or rows will be returned or displayed, based on the criteria described after the keyword \textbf{where}. If we want to repeatedly apply the same algorithm (sequence of steps) to the different elements (values) with just one loop, we then define an \textbf{array} of the type a(n), we initialize our variables, and we are done!

So, if we are asked, for instance, to create a program in fortran performing a pocket-calculator, we may write:

\begin{verbatim}
program calculator
implicit none
real:: a, b, sum, sub, prod, ratio
Integer:: choice
Read(*,*) choice
if (choice = =) 1 then
  sum = a+b
  write(*,*) sum
elseif (choice = = 0) then
  sub=a-b
  write(*,*) sub
end if
if (choice = =3) then
  prod=a*b
  write(*,*) prod
elseif (choice = = 2) then
  ratio=a/b
  write(*,*) ratio
end if
end
\end{verbatim}

Of course the program above is just another way of saying the calculator: “The programs begins with a name, ‘calculator’, and ends up when I say ‘end’; it’s me who decides which are the variables to be used, so do not take anything for granted (implicit none). Namely, they are sum (addi-
tion), sub (subtraction), prod (product) and ratio and of course they have to be real; then I’m going to use 4 integers for the 4 basic Maths operation, and they deal with my choice with respect to the Maths I’ll be doing –so it’s you that ought to ask about that. So if I choose 1 you’ll perform an addition and then you’ll write the result; otherwise if I choose 0 you’ll do and write the subtraction. In the same way 3 is going to correspond to the product and 2 will show a ratio.

If we are asked to write a program calculating the instantaneous displacement and falling time of a projectile ball we can write

program projectile / implicit none / real:: g, t, vx, vy, x, y / x=0 / y=0 / t=0 / g=9.8 / vx=5/ vy=5/ do while (y>=0) / t=t+0.01 / x = vx*t / y = Vx * t-1./2. *g*t**2 / if (y>=0) then / write(*,*)  'at the time t =', t, 'x=', x, 'y =', y /end if /end do /end

**TEXT BUILDING:** write the corresponding text.

**EXERCISE:** Use any IT program and convert it into a coherent text using appropriate clauses to describe the loops. If you wish you may use the program below about definite integrals.

---

**DATABASES: an example of structured language**

Let’s read the following text from http://www.sqlcourse.com/intro.html:

SQL stands for Structured Query Language. SQL is used to communicate with a database. SQL statements are used to perform tasks such as update data on a database, or retrieve data from a database. The standard SQL commands such as "Select", "Insert", "Update", "Delete", "Create", and "Drop" can be used to accomplish almost everything that one needs to do with a database. A relational database system contains one or more objects called tables. The data or information for the database are stored in these tables. Tables are uniquely identified by their names and are comprised of columns and rows. Columns contain the column name, data type, and any other attributes for the column. Rows contain the records or data for the columns. A **select** statement is used to query the database and retrieve selected data that match the criteria that you specify. The column names that follow the select keyword determine which columns will be returned in the results. You can select as many column names that you’d like, or you can use a "*" to select all columns. The table name that follows the keyword **from** specifies the table that will be queried to retrieve the desired results. The **where** clause (optional) specifies which data values or rows will be returned or displayed, based on the criteria described after the keyword **where**. Conditional selections used in the **where** clause are  = , >, < , >=, <=, < > (not equal to) and LIKE. The LIKE
pattern matching operator can also be used in the conditional selection of the where clause. Like is a very powerful operator that allows you to select only rows that are "like" what you specify. The percent sign "%" can be used as a wild card to match any possible character that might appear before or after the characters specified. For example:

<table>
<thead>
<tr>
<th>Sample Table: empinfo</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>first</strong></td>
</tr>
<tr>
<td>John</td>
</tr>
<tr>
<td>Mary</td>
</tr>
<tr>
<td>Eric</td>
</tr>
<tr>
<td>Mary Ann</td>
</tr>
<tr>
<td>Ginger</td>
</tr>
<tr>
<td>Sebastian</td>
</tr>
<tr>
<td>Gus</td>
</tr>
<tr>
<td>Mary Ann</td>
</tr>
<tr>
<td>Erica</td>
</tr>
<tr>
<td>Leroy</td>
</tr>
<tr>
<td>Elroy</td>
</tr>
</tbody>
</table>

Select statement exercises

1) Write the corresponding text to the following queries:

```sql
select first, last, city from empinfo;
select last, city, age from empinfo
    where age > 30;
select first, last, city, state from empinfo
    where first LIKE 'J%';
select * from empinfo;
select first, last, from empinfo
    where last LIKE '%s';
select first, last, age from empinfo
    where last LIKE '%illia%';
select * from empinfo where first = 'Eric';
```

2) Write the corresponding SQL queries to the following statements:

1. Display the first name and age for everyone that's in the table.
2. Display the first name, last name, and city for everyone that's not from Payson.
3. Display all columns for everyone that is over 40 years old.
4. Display the first and last names for everyone whose last name ends in an "ay".
5. Display all columns for everyone whose first name equals "Mary".
6. Display all columns for everyone whose first name contains "Mary".

**EXERCISE**

*Look at the following tables:*

**items_ordered**

<table>
<thead>
<tr>
<th>customerid</th>
<th>order_date</th>
<th>item</th>
<th>quantity</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>10330</td>
<td>30-Jun1999</td>
<td>Pogo stick</td>
<td>1</td>
<td>28.00</td>
</tr>
<tr>
<td>10101</td>
<td>30-Jun1999</td>
<td>Raft</td>
<td>1</td>
<td>58.00</td>
</tr>
<tr>
<td>10298</td>
<td>01-Jul-1999</td>
<td>Skateboard</td>
<td>1</td>
<td>33.00</td>
</tr>
<tr>
<td>10101</td>
<td>01-Jul-1999</td>
<td>Life Vest</td>
<td>4</td>
<td>125.00</td>
</tr>
<tr>
<td>10299</td>
<td>06-Jul-1999</td>
<td>Parachute</td>
<td>1</td>
<td>1250.00</td>
</tr>
<tr>
<td>10339</td>
<td>27-Jul-1999</td>
<td>Umbrella</td>
<td>1</td>
<td>4.50</td>
</tr>
<tr>
<td>10449</td>
<td>13-Aug1999</td>
<td>Unicycle</td>
<td>1</td>
<td>180.79</td>
</tr>
<tr>
<td>10439</td>
<td>14-Aug1999</td>
<td>Ski Poles</td>
<td>2</td>
<td>25.50</td>
</tr>
<tr>
<td>10101</td>
<td>18-Aug1999</td>
<td>Rain Coat</td>
<td>1</td>
<td>18.30</td>
</tr>
<tr>
<td>10449</td>
<td>01-Sep1999</td>
<td>Snow Shoes</td>
<td>1</td>
<td>45.00</td>
</tr>
<tr>
<td>10439</td>
<td>18-Sep1999</td>
<td>Tent</td>
<td>1</td>
<td>88.00</td>
</tr>
<tr>
<td>10298</td>
<td>19-Sep1999</td>
<td>Lantern</td>
<td>2</td>
<td>29.00</td>
</tr>
<tr>
<td>10410</td>
<td>28-Oct1999</td>
<td>Sleeping Bag</td>
<td>1</td>
<td>89.22</td>
</tr>
<tr>
<td>10438</td>
<td>01-Nov1999</td>
<td>Umbrella</td>
<td>1</td>
<td>6.75</td>
</tr>
<tr>
<td>10438</td>
<td>02-Nov1999</td>
<td>Pillow</td>
<td>1</td>
<td>8.50</td>
</tr>
<tr>
<td>10298</td>
<td>01-Dec1999</td>
<td>Helmet</td>
<td>1</td>
<td>22.00</td>
</tr>
<tr>
<td>10449</td>
<td>15-Dec1999</td>
<td>Bicycle</td>
<td>1</td>
<td>380.50</td>
</tr>
<tr>
<td>10449</td>
<td>22-Dec1999</td>
<td>Canoe</td>
<td>1</td>
<td>280.00</td>
</tr>
<tr>
<td>10101</td>
<td>30-Dec1999</td>
<td>Hoola Hoop</td>
<td>3</td>
<td>14.75</td>
</tr>
<tr>
<td>10330</td>
<td>01-Jan2000</td>
<td>Flashlight</td>
<td>4</td>
<td>28.00</td>
</tr>
<tr>
<td>10101</td>
<td>02-Jan2000</td>
<td>Lantern</td>
<td>1</td>
<td>16.00</td>
</tr>
<tr>
<td>10299</td>
<td>18-Jan2000</td>
<td>Inflatable Mattress</td>
<td>1</td>
<td>38.00</td>
</tr>
<tr>
<td>10438</td>
<td>18-Jan2000</td>
<td>Tent</td>
<td>1</td>
<td>79.99</td>
</tr>
<tr>
<td>10413</td>
<td>19-Jan2000</td>
<td>Lawnchair</td>
<td>4</td>
<td>32.00</td>
</tr>
<tr>
<td>10410</td>
<td>30-Jan2000</td>
<td>Unicycle</td>
<td>1</td>
<td>192.50</td>
</tr>
<tr>
<td>10315</td>
<td>2-Feb-2000</td>
<td>Compass</td>
<td>1</td>
<td>8.00</td>
</tr>
<tr>
<td>10449</td>
<td>29-Feb2000</td>
<td>Flashlight</td>
<td>1</td>
<td>4.50</td>
</tr>
<tr>
<td>10101</td>
<td>08-Mar2000</td>
<td>Sleeping Bag</td>
<td>2</td>
<td>88.70</td>
</tr>
</tbody>
</table>
### customers

<table>
<thead>
<tr>
<th>customerid</th>
<th>firstname</th>
<th>lastname</th>
<th>city</th>
<th>state</th>
</tr>
</thead>
<tbody>
<tr>
<td>10101</td>
<td>John</td>
<td>Gray</td>
<td>Lynden</td>
<td>Washington</td>
</tr>
<tr>
<td>10298</td>
<td>Leroy</td>
<td>Brown</td>
<td>Pinetop</td>
<td>Arizona</td>
</tr>
<tr>
<td>10299</td>
<td>Elroy</td>
<td>Keller</td>
<td>Snoqualmie</td>
<td>Washington</td>
</tr>
<tr>
<td>10315</td>
<td>Lisa</td>
<td>Jones</td>
<td>Oshkosh</td>
<td>Wisconsin</td>
</tr>
<tr>
<td>10325</td>
<td>Ginger</td>
<td>Schultz</td>
<td>Pocatello</td>
<td>Idaho</td>
</tr>
<tr>
<td>10329</td>
<td>Kelly</td>
<td>Mendoza</td>
<td>Kailua</td>
<td>Hawaii</td>
</tr>
<tr>
<td>10330</td>
<td>Shawn</td>
<td>Dalton</td>
<td>Cannon Beach</td>
<td>Oregon</td>
</tr>
<tr>
<td>10338</td>
<td>Michael</td>
<td>Howell</td>
<td>Tillamook</td>
<td>Oregon</td>
</tr>
<tr>
<td>10339</td>
<td>Anthony</td>
<td>Sanchez</td>
<td>Winslow</td>
<td>Arizona</td>
</tr>
<tr>
<td>10408</td>
<td>Elroy</td>
<td>Cleaver</td>
<td>Globe</td>
<td>Arizona</td>
</tr>
<tr>
<td>10410</td>
<td>Mary Ann</td>
<td>Howell</td>
<td>Charleston</td>
<td>South Carolina</td>
</tr>
<tr>
<td>10413</td>
<td>Donald</td>
<td>Davids</td>
<td>Gila Bend</td>
<td>Arizona</td>
</tr>
<tr>
<td>10419</td>
<td>Linda</td>
<td>Sakahara</td>
<td>Nogales</td>
<td>Arizona</td>
</tr>
<tr>
<td>10429</td>
<td>Sarah</td>
<td>Graham</td>
<td>Greensboro</td>
<td>North Carolina</td>
</tr>
<tr>
<td>10438</td>
<td>Kevin</td>
<td>Smith</td>
<td>Durango</td>
<td>Colorado</td>
</tr>
<tr>
<td>10439</td>
<td>Conrad</td>
<td>Giles</td>
<td>Telluride</td>
<td>Colorado</td>
</tr>
<tr>
<td>10449</td>
<td>Isabela</td>
<td>Moore</td>
<td>Yuma</td>
<td>Arizona</td>
</tr>
</tbody>
</table>

Given the two tables above write an appropriate text-program for the following queries. You must define the SQL statements you are going to use and say why you are using them in that context:

1. From the `items_ordered` table, select a list of all items purchased for customerid 10449. Display the customerid, item, and price for this customer.

2. Select the customerid, order_date, and item values from the `items_ordered` table for any items in the item column that starts with the letter “S”.

3. Select the item and price for all of the items in the `items_ordered` table so that the price is greater than 10.00. Display the results in ascending order based on the price. (Use the keyword “order by” to display the results of your query in either upwards or downwards order).
Given the two following text-program write the corresponding queries in an appropriate English text. Then say if there is any difference between the first two reference tables and the third one.

1. SELECT * FROM items_ordered
   WHERE item = 'Tent';
2. SELECT DISTINCT item
   FROM items_ordered;
3. SELECT customers.customerid, customers.firstname, customers.lastname,
   items_ordered.order_date, items_ordered.item, items_ordered.price
   FROM customers, items_ordered
   WHERE customers.customerid = items_ordered.customerid;

DEFINITE INTEGRALS

We want to write a text in order to figure out, define and quantify a definite integral. Let’s consider the following graphs to focus on motion with constant acceleration

The first graph shows us the position of a material point after a bunch of seconds. The distance function shows that at any seconds we are going a bit further. How do we calculate the instantaneous velocity? – that is, the instant rate of change of distance with respect to time? For instance if the distance $y=16t^2$ then $v=ds/dt=32t$. Thus the second one is the graph of the derivative: it is a line with a slope of 32. We realize the derivative of distance is velocity, the anti-derivative of velocity is distance. Now, let’s assume we are given only the graph of velocity. What is the distance after $t$ seconds? The distance will be equal to $v*t$ – that is, the product of velocity by time. Now, in order to go on, let’s divide the area under the curve $v(t)$ into equally partitioned rectangles, as explained below, of base $t$. Let’s call these small time fragments $dt$, which, emphasizing an infinitely small change in time, tell us how much time is goes by. Over this small change in time we have a constant velocity. Actually, each rectangles represent our constant velocity. The distance that this object travels over the small time will be the product of that small times by its velocity – that is by the width of this distance. If we take the change in time (the base of the rectangle) and we multiply it by the velocity (the height of the rectangle) we will figure
out the area of that same infinitely skinny rectangle. How far do we travel after \( t_0 \) (\( t \) sub-not) seconds? As we draw other rectangles so as to cover the entire area, as we make the \( dt \) smaller and smaller, we have more and more rectangles, so that the approximation is better. Each of the rectangles have an equal width, they are equally partitioned between two boundaries, say \( a \) and \( b \), and the area of the rectangle was the area evaluated at the left-end point of each rectangle. If we generalize this in sigma notation we can define, using Riemann’s notation, the integral of a generic function \( f \) (in our case \( v \)) in the interval \([a, b]\) as the limit as \( n \) (the number of rectangles) approaches infinity of the following integral sum:

\[
\sigma_n = \frac{b - a}{n} \sum_{k=1}^{n} f(t_k)
\]

which is known as Riemann’s sum. If this limit exists, if it is a finite limit and if it does not depend on the choice of the points \( t_k \), we can write:

\[
\int_a^b f(x) \, dx = \lim_{n \to +\infty} \sigma_n = \lim_{n \to +\infty} \frac{b - a}{n} \sum_{s=1}^{n} f(t_s)
\]

In other words we can represent our partition as the limit as \( n \) approaches infinity of the sum from \( k=1 \) to infinity of the function \( v(t_{k-1}) \) (\( v \) sub-k minus 1 because we start counting from 0) times \( \Delta x \), where \( \Delta x \) is equal to \((b-a)/n\), which is equal to the integral from \( a \) to \( b \) of \( v(t_{k-1}) \).

This is a particular instance of Riemann’s sums, which is one of the mainstream forms and rigorous definition of what an integral is. Of course, as \( n \) approaches infinity we better approximate the area. As \( n \to \infty \), \( \Delta x \to dt \), thus the limit of the sum equals the integral from \( a \) to \( b \) of \( v(t) \)\( \times dt \), which allows us to figure out the exact change in position between \( a \) and \( b \). Hence this integral is equal to \( y(b) - y(a) \), where \( y \), according to our notation, is the displacement. This is one of the applications of the second fundamental theorem of calculus, where \( y(t) \) is the anti-derivative of \( v(t) \).

For all these reason we can rewrite about the graphs above as follows:
If the acceleration of an object is time dependent, then calculus methods are required for motion analysis. The relationships between position, velocity and acceleration can be expressed in terms of derivatives or integrals.

If the acceleration of an object is time dependent, then calculus methods are required for motion analysis. The relationships between position, velocity and acceleration can be expressed in terms of derivatives or integrals.

\[
y = \int v \, dt
\]
\[
= \int (v_0 + at) \, dt
\]
\[
y = y_0 + v_0 t + \frac{1}{2} at^2
\]
Integrate velocity to get position

\[
v = \int a \, dt = v_0 + at
\]
Integrate acceleration to get velocity

\[
a = \text{constant}
\]

\[
\frac{dv}{dt} = \frac{d^2 r}{dt^2}
\]

Motion relationships in one dimension.

\[
y = y_0 + v_0 t + \frac{1}{2} at^2
\]
Derivative of position is velocity

\[
v = \frac{dy}{dt}
\]

\[
v = v_0 + at
\]
Derivative of velocity is acceleration

\[
a = \frac{dv}{dt} = a
\]

<table>
<thead>
<tr>
<th>Derivative Form</th>
<th>Integral Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>( r(t) )</td>
</tr>
<tr>
<td>Velocity</td>
<td>( v(t) = \frac{dr}{dt} )</td>
</tr>
<tr>
<td>Acceleration</td>
<td>( a(t) = \frac{dv}{dt} = \frac{d^2 r}{dt^2} )</td>
</tr>
</tbody>
</table>

**TEXT BUILDING:** write a text consistent with the graphs and the schemes above.
We can now write about definite integrals more thoroughly and connect our result to some programming language as follows.

Now read the following material about integrals to be found at http://www.mathcs.emory.edu/~cheung/Courses/170/Syllabus/07/rectangle-method.html and analyze the figures, focus on the programs, then write a coherent text using active/passive forms, appropriate tenses and if clauses 1,2,3.

- A definite integral \( \int_a^b f(x) \, dx \) is the integral of a function \( f(x) \) with fixed end point \( a \) and \( b \):
  - The integral of a function \( f(x) \) is equal to the area under the graph of \( f(x) \).
  - Graphically explained:

  ![Graphically explained](image)

- **Rectangle Method:**
  - The rectangle method (also called the midpoint rule) is the simplest method in Mathematics used to compute an approximation of a definite integral.

**Rectangle Method: explained**
- Divide the interval \([a..b]\) into \( n \) pieces; each piece has the same width:
The width of each piece of the smaller intervals is equal to:

\[
\frac{b - a}{n}
\]

The definite integral (= area under the graph is approximated using a series of rectangles:

The area of a rectangle is equal to:

\[
\text{area of a rectangle} = \text{width} \times \text{height}
\]

We (already) know the width of each rectangle:

\[
\frac{b - a}{n}
\]

We still need to find the height of each rectangle.
The different rectangles have different heights.

Heights of each rectangle:
- The heights of each rectangle = the function value at the start of the (small) interval.

Example: first interval
- First (small) interval: \([a .. (a + width)]\) (remember that: \(width = (b - a)/n\))
- Height of first (small) interval:
  - Therefore: height of first rectangle = \(f(a)\)
  - Area of the rectangle = \(f(a) \times width\)

Example: second interval
- The second (small) interval is \([(a+width) .. (a+2width)]\) (remember that: \(width = (b - a)/n\))
- Height of first (small) interval:
Therefore:

- **height** of first rectangle = \( f(a+width) \)
- Area of the rectangle = \( f(a+width) \times width \)

Example: third interval

- The third (small) interval is \([ (a+2width) .. (a+3width) ] \) (remember that: \( width = (b - a)/n \))
- **Height** of first (small) interval:
Therefore: **height** of first rectangle = \( f(a+2\times\text{width}) \)

We see a **pattern** emerging:

- **Height of** rectangle 1 = \( f(a + 0\times\text{width}) \)
- **Height of** rectangle 2 = \( f(a + 1\times\text{width}) \)
- **Height of** rectangle 3 = \( f(a + 2\times\text{width}) \)
- ...  
- **Height of** rectangle \( n-1 \) = \( f(a + (n-2)\times\text{width}) \)
- **Height of** rectangle \( n \) = \( f(a + (n-1)\times\text{width}) \)

Note: there are \( n \) (smaller) interval in total.

**Conclusion:**

\[
\text{Height of rectangle } i = f( a + (i-1)\times\text{width} ) = \text{the function value at the point } "a + (i-}
Recall that:

\[
\text{width} = \frac{b - a}{n}
\]

- The area of the rectangles:

This figure helps you to visualize the computation:

- The width of rectangle \( i \) is equal to:

\[
\text{width} = \frac{b - a}{n}
\]

- The height of rectangle \( i \) is equal to:

\[
\text{height} = f(a + (i-1) \times \text{width})
\]

- The area of rectangle \( i \) is equal to:

\[
\text{area} = \text{width} \times \text{height}
\]
o The approximation of the definite integral:

\[
\text{Approximation} = \text{sum of the area of the rectangles} = \text{area of rectangle 1} + \text{area of rectangle 2} + \ldots + \text{area of rectangle } n = \text{width} \times f(a + (1-1) \times \text{width}) + \text{width} \times f(a + (2-1) \times \text{width}) + \ldots + \text{width} \times f(a + (n-1) \times \text{width})
\]

• The general running sum algorithm
  o We have seen a running sum computation algorithm previously that adds in simple series of numbers:

    Compute the sum: \( 1 + 2 + 3 + \ldots + n \)
    Running sum algorithm:

    \[
    \text{sum} = 0; \quad // \text{Clear sum}
    \text{for} \ ( i = 1; i <= n ; i++ ) \ \\
    \{ \quad \text{sum} = \text{sum} + i; \quad // \text{Add } i \text{ to sum} \ \\
    \}
    \]

  o The running sum algorithm can be generalized to add a more general series

    Example: compute \( 1^2 + 2^2 + 3^2 + \ldots + i^2 + \ldots + n^2 \)

    The \( i^{th} \) term in the sum = \( i^2 \)
    Therefore, the running sum algorithm to compute this sum is:

    \[
    \text{sum} = 0; \quad // \text{Clear sum}
    \text{for} \ ( i = 1; i <= n ; i++ ) \ \\
    \{ \quad \text{sum} = \text{sum} + i*i; \quad // \text{Add } i^2 \text{ to sum} \ \\
    \}
    \]
• **Algorithm to compute the sum of the area of the rectangles**
  o We can use the **running sum algorithm** to compute the *sum of the area of the rectangles*
  
  o Recall:
    
    \[
    \text{Aproximation} = \text{sum of the area of the rectangles}
    = \text{width} \times f(a + (1-1)\times\text{width}) \\
    + \text{width} \times f(a + (2-1)\times\text{width}) \\
    + \ldots \\
    + \text{width} \times f(a + (i-1)\times\text{width}) \\
    + \ldots \\
    + \text{width} \times f(a + (n-1)\times\text{width})
    \]
    
    Where:
    
    \[
    \text{width} = (b - a) / n
    \]
  
  o The *ith term* of the **running sum** is equal to:
    
    \[
    \text{i}^{th} \ \text{term} = \text{width} \times f(a + (i-1)\times\text{width})
    \]
  
  o **Algorithm** to compute the *sum* of the area of the rectangles:

    ```java
    Variables:
    
    double w;        // w contains the width
    double sum;      // sum contains the running sum

    Algorithm to compute the sum:
    
    w = (b - a)/n;   // Compute width
    sum = 0.0;       // Clear running sum
    
    for ( i = 1; i <= n; i++ )
    {
        sum = sum + w*f(a + (i-1)*w);
    }
    ```

• **The Rectangle Method in Java**
  o **Rough algorithm** (pseudo code):
input a, b, n; // a = left end point of integral
      // b = right end point of integral
      // n = # rectangles used in the approximation

Rectangle Method:

w = (b - a)/n; // Compute width

sum = sum of area of the n rectangles; // Compute area

print sum; // Print result

Algorithm in Java:

```java
public class RectangleMethod01
{
    public static void main(String[] args)
    {
        double a, b, w, sum, x_i;
        int i, n;

        **** Initialize a, b, n ****

        /* -------------------------------------------
--------
The Rectangle Rule Algorithm
-------------------------------------------
-------- */

        w = (b-a)/n; // Compute width
        sum = 0.0; // Clear running sum

        for ( i = 1; i <= n; i++ )
        {
            x_i = a + (i-1)*w; // Use x_i to simplify formula...
            sum = sum + ( w * f(x_i) ); // width * height of rectangle i
        }
    }
}
```
Example 1: compute \( \int_0^1 x^3 \, dx \) (the exact answer = 0.25)

```java
public class RectangleMethod01
{
    public static void main(String[] args)
    {
        double a, b, w, sum, x_i;
        int i, n;

        a = 0.0; b = 1.0; // \( \int_0^1 x^3 \, dx \)
        n = 1000; // Use larger value for better approximation

        w = (b-a)/n; // Compute width
        sum = 0.0; // Clear running sum

        for ( i = 1; i <= n; i++ )
        {
            x_i = a + (i-1)*w;
            sum = sum + ( w * (x_i * x_i * x_i) ); // \( f(x_i) = (x_i)^3 \)
        }

        System.out.println("Approximate integral value = " + sum);
    }
}
```

Example Program: (Demo above code)

- Prog file: [click here](#)

How to run the program:
Right click on link and save in a scratch directory

To compile: javac RectangleMethod01.java

To run: java RectangleMethod01

Output: Approximate integral value = 0.2495002499999998
Exact answer: 0.25

Example: compute \( \int_{1}^{2} \frac{1}{x} \, dx \)  (the answer = \( \ln(2) \))

```java
public class RectangleMethod02 {
    public static void main(String[] args) {
        double a, b, w, sum, x_i;
        int i, n;
        a = 1.0; b = 2.0;  // 1 \int_{1}^{2} \frac{1}{x} \, dx
        n = 1000;          // Use larger value for better approximation
        w = (b-a)/n;                      // Compute width
        sum = 0.0;                        // Clear running sum
        for ( i = 1; i <= n; i++ ) {
            x_i = a + (i-1)*w;
            sum = sum + ( w * (1/x_i) );   // f(x_i) = 1/x_i
        }
        System.out.println("Approximate integral value = " + sum);
    }
}
```

Example Program: (Demo above code)
How to run the program:

- Right click on link and save in a scratch directory
- To compile: `javac RectangleMethod02.java`
- To run: `java RectangleMethod02`

Output: Approximate integral value = 0.6933972430599376
Exact answer: \( \ln(2) = 0.69314718 \)

The effect of the number of rectangles used in the approximation

- Difference in the approximations when using different number of rectangles:

  - Clearly:
    - Using more rectangles will give us a more accurate approximation of the area under the graph

  However:
  - Using more rectangles will make the algorithm add more numbers
    I.e., the algorithm must do more work --- it must add more smaller values (because the rectangles are smaller and have smaller areas)

- Trade off:
  - Often, in computer algorithms, a more accurate
result can be obtained by a longer running execution of the same algorithm.

We call this phenomenon: trade off

You cannot gain something without giving up on something else

You can experience the trade off phenomenon by using \( n = 1000000 \) in the above algorithm.

It will run slower, but give you very accurate results!

Outputs for \( n = 1000000 \):

- Approximate integral value of \( \int_0^1 x^3 \, dx = 0.24999950000025453 \)
- Approximate integral value of \( \int_1^2 (1/x) \, dx = 0.6931474305600139 \)

---

COHESION AND COHERENCE OF A MECHANICAL SYSTEM

In text building cohesion is essentially determined by inter-sentential relationships, whereas coherence is based on semantic relationships. Otherwise we can say that coherence is achieved through syntactical features such as the use of deictic or a logical tense structure, as well as presuppositions and implications connected to general world knowledge. The purely linguistic elements that make a text coherent are subsumed under the term cohesion. The same logic is used in the building of Mathematical formalisms and is paramount in logical sequences such as equations. Let’s read the following text.

Dealing with mechanics applied to machines means dealing with power transmission. Actually, mechanical power is the rate at which work is being provided once the system is in movement. In symbols:

\[
P = \frac{dW}{dt} = F ds/dt = Fv,
\]

where, again, \( P \) is the power, \( W \) is the work done, \( F \) is the exerted force, \( Ft \) its tangent component, \( s \) is the movement in space (the displacement), \( v \) is the velocity. We read this equation as follows: \( P \) is equal to (or equals) \( W \) over (or divided by) \( t \), or, more thoroughly, the ratio of the (infinitely small, or infinitesimal) work done to time, which is equal to \( F \) times (or multiplied by) \( s \) all over \( t \), which in turn is equal to \( F \) times \( v \), where the work done is the integral on a curve (say from a point \( a \) to a point \( b \)) of the product of the force by the displacement, which is equal to the same integral of the product of the same force by the cosine of an angle (that is, the ratio of the adjacent side of a right triangle over its hypothenuse) by the
same displacement, which finally equals the integral of the tangent component of that force by that displacement. On a linguistic level, an equation is a logical expression in which the operators work as connectives. In other words, it is a way of matching two or more definitions. Let’s do another example. Changing the words so that the meaning stays the same, we can define work, generally speaking, as energy transferred by force and we can define energy as the ability to do work. To put it into an equation we write, in the simplest possible way, \( W = Fs \). It means that if we apply a force of 10 N (N=Newton) on, say, a massive block and we move that block 7 m (m=meter), then the work done is both known and expressed by the equation \( W=10N \times 7m = 70Nm = 70J \), where J=Joules=Nm=Newtons times meters=Newtons multiplied by meters= the unit of measurement of energy. Consequently (and that is a connective!), Nm is also a way to describe work and it is equal to one Joule, a way to describe energy. Useless to say, we can write or read the symbol = with connectives like: which equals, which brings to, so, so that, as a result of and the like. Physically speaking, however, power is nothing else than a sent unit of work per second.

A simple example of power transmission between two shafts not too distant from one another is that of friction wheels. The figure below shows the scheme of such a transmission. We have two wheels whose diameters are D1 and D2. The first one is placed on the driving shaft and has an angular velocity \( \omega_m \) and a torque \( M_m \). The second one is on the driven shaft and has an angular velocity \( \omega_u \) and a torque \( M_r \).

Motion can be transmitted thanks to a tangent force \( T \) because of the force \( R \) that pushes the wheels one against the other. If \( f \) is the friction coefficient, we have \( T = f R \). Namely, we read the formula as follows: \( T \) is equal to (or equals) \( f \) times \( R \) – that is, the force is equal to the product of \( f \) by \( R \). On a physical level, it means that \( T \) is proportional to \( R \) through (or “via”) \( f \): the higher \( f \), the stronger the tangent force \( T \); the lower \( f \), the weaker the force.

**Text building**: What do we use friction wheels for? How do they work?

If we call \( I \) the distance shown in the figure below we get \( I = \frac{1}{2}(D_1 + D_2) \).
If the wheels do not slide, the velocity of the contact point on wheel 1 will equal the velocity of the contact point on wheel 2, so that $V_1 = V_2$. Consequently $\frac{1}{2}(\omega_1 D_1) = \frac{1}{2}(\omega_2 D_2)$. That is to say:

$$ i = \frac{\omega_1}{\omega_2} = \frac{D_2}{D_1} $$

The transmission ratio $i$ depends on the diameter of the two wheels. This last equation, together with that of the interaxes $I$, makes us size the diameters of the two wheels correctly.

On a practical level, the uses of friction wheels are rather limited, though being silent and having a regular transmission, for we can use them only at low powers. For high powers the force $T = fR$ must be elevated, but since the friction coefficient for commonly used materials such as steel or cast iron is rather low ($f=0,10-0,15$), there should be very high pushing forces $R$ so that shafts, pins, bearings, etc, would be strongly stressed.

As we have just said, friction wheels are limited for what concerns transmission of high powers, because of the huge radial stresses they must undergo in order to ensure their adherence. Starting from two ideal friction wheels, we can obtain a series of cogs on their external surfaces – that is, a series of projections on the edge of a wheel transferring motion by engaging with another series - alternating with empty spaces, that while in motion are easily interpenetrated; in this case, the transmission of power is no longer due to the friction but to the “pushing” force that each cog of the driving wheel exerts on those of the driven wheel. In this way, provided that the built cogs are strong enough, we can transmit high powers.

Actually, it is possible to convert rotational motion into linear (translational) motion using the mechanism pinion/rack, where the pinion’s rotational motion is converted into a translational motion by the rack. Given a gear, we define the pinion as the cogwheel with the smallest diameter and the wheel as that with the largest diameter. The interaxes is the distance between the axis of the two wheels.

If $\omega_1$ is the angular velocity of the pinion and $\omega_2$ the angular velocity of the wheel, then we define the transmission ratio as usual $i = \omega_1 / \omega_2$

Word builder: show the pinion (“pignone”) and the rack (“cremagliera”) in the figure below.
A striking example of this is the car’s steering-wheel mechanism. While driving, the steering’s rotation is being converted into the translation of the elements acting upon the wheels.

**TEXT BUILDING:** Describe how this system works with respect to your own car: what happens if you have to turn right, left, or get straight on?

As shown in the figure below, the force transmitted from the driving wheel to the driven one is the tangent component $F_t$ of the “pushing” force $F$. This component lies upon the tangent to the two primitive circumferences.
For this tangent component of the force we have \( F_t = F \cos \theta = \frac{C_m}{R_1} = \frac{C_r}{R_2} \), where \( C_m \) is the driving ‘pair’ (‘coppia motrice’) and \( C_r \) is the resisting ‘pair’ (‘coppia resistente’). Namely, the ‘pairs’ obviously refer to the torque. Of course the radial component \( F_r \) is not responsible for motion and constitutes a solicitation all over the shaft on which the wheels are keyed. Its module is \( F_r = F \sin \theta \).

This suggests that we ought to render the pressure angle \( \theta \) very small in order to increase the value of the ‘useful’ force \( F_t \). In the construction of cogwheels, one cannot go below a certain number of cogs without compromising the proper functioning of the system. The value of the angle of pressure affects the minimum number of cogs that a wheel can have in order to conjugate the cog’s profile. In practice, we assign the number of cogs as a function of the pressure angle and of the transmission ratio using the following formula:

\[
Z_{\text{min}} = \frac{2}{i^2 + (1 + 2i) \sin^2 \theta - i}^{1/2},
\]

where \( i \) is the transmission ratio.

As for the minimum cogs’ number (\( Z_{\text{min}} \)) as a function of \( \theta \) and \( i \) we have the following table:

<table>
<thead>
<tr>
<th>( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta = 15^\circ )</td>
<td>21</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>28</td>
<td>29</td>
<td>29</td>
<td>29</td>
<td>29</td>
</tr>
<tr>
<td>( \theta = 20^\circ )</td>
<td>13</td>
<td>15</td>
<td>15</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>17</td>
<td>17</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>( \theta = 25^\circ )</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
</tr>
</tbody>
</table>

**TEXT BUILDING**

Write a text using the table above and the three if clauses.

Ex: given \( i = 1 \), if the pressure angle is equal to \( 15^\circ \) then the transmission ratio will be equal to 21; if it were equal to \( 20^\circ \), then \( i \) would be equal to 13; if it had been equal to \( 25^\circ \), then \( i \) would have been equal to 9.

Use the following verbs: measure, span, be wide, equal, have the value (of), correspond (to), show an amplitude (of). Then re-write the sentences using the transmission ratio in the if clause and the pressure angle in the main clause.

Use the following verbs in the negative form: be equal to, equal, approach, approximate, result, be given as.
Then write down appropriate questions and answers like “what would the angle be equal to if….?” using the three if clauses.
Finally use unless, when, as soon as, and write the corresponding logical three-type if clauses.

**TEXT BUILDING:**
Write a complete text using all the information above and appropriate relative clauses and passive forms.

**Remember:**
Qualitative description of the system:
The torque is transmitted by the driving wheel
System in movement:
The torque is being transmitted by the driving wheel

**THE ROD/CRANK SYSTEM**
Adapted from http://www.istitutopesenti.it/dipartimenti/meccanica/meccanica/biella.pdf

The system is used to convert an alternating linear motion into a rotational motion (or vice versa). It is used in most endothermic engines and in volumetric machines (pumps, compressors). Its main components are:
- The PLUNGER (or PISTON) that leads inside the cylindrical pin on which is articulated the top end of the rod (the connecting rod’s foot), while the lower end (the connecting rod’s head) embraces the pin placed at the end of the crank keyed on the crankshaft of the engine;
- The CONNECTING ROD, that is to say a rod connected with two hinges, on one side to the plunger, on the other side to the crank.
- The CRANK, a rod connected with the connecting rod and constrained to rotate around the point O;
- The FRAME which is the medium on which the system rests.

Word building. Match the English words in the text with the Italian words in the picture.
Text building. Write a coherent text using the pieces of information above.
By turning the crank, the piston will move with variable speed along a straight line between two extreme points called TOP DEAD POINT (TDP) and LOW DEAD POINT (LDP). These points are called dead points because here the piston’s velocity is equal to zero.

**Text building**: write a coherent text describing the system below and the pieces of information describing it in the next page.

The kinematic study is mainly devoted to the determination of the instantaneous velocity and acceleration of the piston (point P in the figure).
- The point P is called CONNECTING ROD’S FOOT
- The point B is called CONNECTING ROD’S HEAD if considered as belonging to the connecting rod
- The point B is called CRANKPIN if considered as belonging to the crank

Let's consider the crank in a general position and denote by
- L the length of the connecting rod
- r the length of the crank that coincides with the radius of the circle described by the point B during the rotation of the crank
- α the central angle formed by the crank and the segment joining the points O and P
- β the angle of inclination of the connecting rod with the segment joining the same points
- s the general shift of the rod’s foot (point P) measured from the position identified by the point M (TDP).

The piston's stroke is equal to 2r –that is, twice the crank’s length.

We assume that the angular velocity of the crank remains constant in time: $\omega = \text{constant}$. It is clear that during the rotation of the crank, as time goes by, the values of s, a, b, will vary, so the values taken ARE NOT CONSTANT, BUT VARY OVER TIME.

Under the assumptions of constant angular velocity for the crank ($\omega = \text{constant}$), for point B (CRANKPIN/ROD’S HEAD) we can now say that:
- Its speed is constant in module: $v = \omega \times r \text{ (m/s)}$
- The direction of this speed varies and is always tangent at a point over the circle and its direction depends on the crank's rotational motion.
- The tangent acceleration is zero because the magnitude of the velocity is constant $\Rightarrow \ddot{v} = 0$
The centripetal acceleration is not equal to zero, because the direction of the velocity is variable: \( Ac = \frac{v^2}{r} \text{[m/s}^2\text{]} \).

Using the more detailed figure below

\[
\begin{align*}
\text{s} & = \text{MP} = \text{MO} - \text{PO} \\
\text{s} & = L + r \\
\text{PO} & = PS + SO = L \cdot \cos \beta + r \cdot \cos \alpha \\
\text{Thus} & \\
\text{s} & = r \cdot (1 - \cos \alpha) + L \cdot (1 - \cos \beta) \\
\text{that is} & \\
\text{s} & = r \cdot (1 - \cos \alpha + \frac{L}{r} - L \cos \beta/r) \\
\text{Given } \mu = \frac{L}{r} \text{ we get} & \\
\text{s} & = r \cdot (1 - \cos \alpha + \mu - \mu \cdot \cos \beta) \\
\text{where we figure out the angles } \alpha \text{ and } \beta \text{ defining, instant by instant, the rod's and crank's} \\
\text{These two angles are not independent, so that we can express } \beta \text{ as a function of } \alpha. \\
\text{As a matter of fact, from the triangle OSB we obtain } BS = r \cdot \sin \alpha \text{ and from PSB we get } BS = L \cdot \sin \beta \\
\text{Thus } r \cdot \sin \alpha = L \cdot \sin \beta \Rightarrow \sin \beta = \left( \frac{L}{r} \right) \sin \alpha = \sin \alpha / \mu \\
\text{But we know that } \sin^2 \beta + \cos^2 \beta = 1 \Rightarrow \cos \beta = (1 - \sin^2 \beta)^{1/2} = (1 - \sin^2 \alpha / \mu)^{1/2} = 1 / \mu (\mu^2 - \sin^2 \alpha)^{1/2} \\
\text{Substituting we get} & \\
\text{s} & = r \cdot (1 - \cos \alpha + \mu - \mu \cdot \cos \beta) = r \cdot (1 - \cos \omega t + \mu - (\mu^2 - \sin^2 \omega t)^{1/2})
\end{align*}
\]
where $\alpha = \omega \cdot t$, being $\omega = \text{constant}$. That’s the expression we were looking for.

The derivative of $s$ with respect to time gives us the velocity of the piston:

$$\frac{ds}{dt} = v = \frac{d}{dt}[r \cdot (1 - \cos \omega t + \mu - (\mu^2 - \sin^2 \omega t)^{1/2})]$$

$$= \omega r (\sin \omega t + \sin(2\omega t) / 2(\mu^2 - \sin^2 (\omega t))^{1/2})$$

whose essential graph is as follows

![Graph](image)

Complete the graph adding the missing equations, points etc. Where is the ‘run’ of the piston? Where are the dead points?

Each value of the velocity is made of two terms, whose graphs are shown in this velocity diagram AS A FUNCTION OF THE CRANK’S ANGLE. Adding point by point the values of the ordinates of the two graphs we get the actual velocity of the rod’s foot. Note that the speed is maximum, for an instant, in a point that is before half the stroke, for the forward stroke, and after half of the stroke for the backward one, while it is zero at the dead points.

**Text building:** write a coherent and cohesive text explaining the kinematic of the system and writing, in a detailed way, the corresponding words and connectives of the equations. Then describe the graph above using your own words and find out any other mechanism working like that.

The derivative of $v$ with respect to time gives us the acceleration of the point:

$$a = \frac{dv}{dt} = \omega^2 \cdot r (\cos \omega t + \cos 2 \omega t / \mu)$$
Complete the graph adding the missing equations, points etc.

Each value is composed of two terms, whose graphs are shown in the diagram as a function of the crank's angle.

We get the instantaneous value of the acceleration of the piston by adding point by point the values of the ordinates of the two graphs.

Notice that the acceleration gets the maximum values at the dead points, while it vanishes at the points at which the velocity is maximum.

Of course it is a tangent acceleration, as being generated by a variation of the module of the velocity or by a change in verse and not by a change in direction, because the direction of velocity remains always constant.

We understand that the acceleration is maximum at the dead points, because at such points the motion is being reversed. Therefore the maximum velocity variation occurs at the dead points.

**Text building**: describe the graph above using your own words and find out a mechanism working like that.
MODALS

General rule:

Present:
Positive: S + modal + base form
Negative: S + modal + not + base form
Interrogative: modal + S + base form

Past:
Positive: S + modal + present perfect
Negative: S + modal + not + present perfect
Interrogative: Modal + S + present perfect

With respect to any systems we deal with, modal verbs allow us to give tips, make predictions, express possibilities or abilities, focus on what we are obliged to do, make deductions.
Tips (advice): should, had better, ought to
Possibility: might (high), may (average), could (remote)
Predictions: will (immediate predictions, opinions), may, might
Obligations, deductions: must
Ability: can
Future in the past: would

Let's read the following text:

IMPLICIT DIFFERENTIATION

When dealing with curves like a circle with equation \( x^2 + y^2 = 1 \), for instance, we cannot explicitly find \( x \) or \( y \), for we have both. In order to find the slope of the tangent line at a point on the circle, say \( P(\sqrt{2}/2, \sqrt{2}/2) \), we'd better calculate the derivative in terms of \( x \) and \( y \) than define \( y \) as a function of \( x \), for that would give us two solutions, namely \( y = \sqrt{1-x^2} \) and \( y = -\sqrt{1-x^2} \). Consequently there must be another way to find the derivative in terms of both. This is called implicit differentiation, and it is an application of the chain rule. More formally, given a differentiable relation \( F(x,y)=0 \), which defines the differentiable function \( y = f(x) \), it might be possible to find the derivative \( f' \) even in the case when we cannot symbolically find \( f \). The relation \( F(x,y)=0 \) is said to define the function \( y = f(x) \) implicitly if, for \( x \) in the domain of \( f \), \( F(x,f(x))=0 \). That means we should apply the operator \( d/dx \) (derivative with respect to \( x \)) on both sides of the equation and solve in terms of the differential \( dy/dx \):

\[
\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(1) = \frac{d}{dx}(2x) + \frac{d}{dy}(y^2)\frac{dy}{dx} = 0 \quad 2x + 2y\frac{dy}{dx} = 0 \quad \frac{dy}{dx} = -\frac{x}{y}
\]

Consequently the slope at \( P \) must equal \(-1\), which means we must have found the value of the derivative in terms of both \( x \) and \( y \).
As it **may** seem clear, implicit differentiation **ought to** recall the chain rule, something we'd **better use** any time we have a composition of functions and we search for the derivative, which tells us the slope at any point along a curve $f$. The following example **might be** clearer: Given the curve

$$(x-y)^2 = x+y+1$$

if we differentiate it implicitly we will **get**

$$\frac{d}{dx} [(x-y)^2] = \frac{d}{dx}(x+y+1)$$

which means

$$2(x - y) (1 - \frac{dy}{dx}) = 1 * \frac{dy}{dx}$$

where $2(x - y) = \frac{d}{d(x-y)} [(x – y)^2]$, that is, the derivative of something squared with respect to that same something; while $(1 – \frac{dy}{dx}) = \frac{d}{dx} (x – y)$, that is, the derivative of something inside the brackets with respect of $x$ and $y$, which is what we are trying to solve for. And this is just the chain rule! –namely, the derivative of the sub-function and the derivative of the entire function.

Finally we get

$$2x – 2y(1 – \frac{dy}{dx}) = 1 + \frac{dy}{dx} \implies \frac{dy}{dx} = \frac{2y-2x+1}{2y-2x-1}$$

Another example of chain rule?

Given $f(x) = \sin^2(x)$, then $\frac{df}{dx} = 2\sin x * \frac{d}{dx}(\sin x) = 2\sin x * \cos x$

Why $2\sin x$? Because we apply the chain rule and we get $\frac{d}{d(\sin x)} [\sin^2(x)] = 2\sin x$

Why $\cos x$? Because $\frac{d}{d(\sin x)} [\sin x] = \cos x$

**EXERCISE:** what is the language function of the modals in bold?

**EXERCISE:** differentiate implicitly $x\sqrt{y}=1$; $\exp(xy^2) = x – y$; $\sin(2x-7y)=16y$; $y = x^{(p/q)}$ building up a text with appropriate modals.
PHOTOSYNTHESIS, RELATIVE CLAUSES AND KERNEL

By combining sentences with a relative clause, your text becomes more fluent and you can avoid repeating certain words. Relative clauses are introduced by a relative pronoun. We use relative clauses to give additional information about something without starting another sentence. If the relative pronoun is followed by a verb, the relative pronoun is a subject pronoun. Subject pronouns must always be used. If the relative pronoun is not followed by a verb (but by a noun or pronoun), the relative pronoun is an object pronoun. Object pronouns can be dropped in defining relative clauses. Defining relative clauses (also called Kernel clauses) give detailed information defining a general term or expression. Defining relative clauses are not put in commas. Non-defining relative clauses (also called non-kernel clauses) give additional information on something, but do not define it. Non-defining relative clauses are put in commas.

Let’s read the following text about LEAF STRUCTURE AND PHOTOSYNTHESIS

Adapted from http://biology.clc.uc.edu/courses/bio104/photosyn.htm

Photosynthesis, which is the process of converting light energy into chemical energy and storing it in the bonds of sugar, occurs in plants and some algae (Kingdom Protista), which need only light energy, CO₂, and H₂O to make sugar. The process of photosynthesis takes place in the chloroplasts that use chlorophyll, the green pigment involved in photosynthesis.

A plant leaf, whose parts include the upper and lower epidermis, the mesophyll, the vascular bundle(s) (veins), and the stomates, is the place where photosynthesis typically happens, while little to none occurs in stems, etc. The upper and lower epidermal cells, which serve primarily as protection for the rest of the leaf, do not have chloroplasts, thus photosynthesis does not occur there. The stomates are holes which occur primarily in the lower epidermis and are for air exchange: they let CO₂ in and O₂ out. The vascular bundles or veins in a leaf are part of the plant’s transportation system, moving water and nutrients around the plant as needed. The mesophyll cells have chloroplasts and this is where photosynthesis occurs. As you hopefully recall, the parts of a chloroplast include the outer and inner membranes, intermembrane space, stroma, and thylakoids stacked in grana. The chlorophyll is built into the membranes of the thylakoids. Chlorophyll looks green because it absorbs red and blue light, making these colors unavailable to be seen by our eyes. It is the green light which is NOT absorbed that finally reaches our eyes, making chlorophyll appear green. However, it is the energy from the absorbed red and blue light that is, thereby, able to be used to do photosynthesis. The green light (that) we can see is not and cannot be absorbed by the plant, and thus cannot be used to do photosynthesis.

Overall Chemical Reaction
The overall chemical reaction involved in photosynthesis is:

\[ 6\text{CO}_2 + 6\text{H}_2\text{O} (+ \text{light energy}) \rightarrow \text{C}_6\text{H}_12\text{O}_6 + 6\text{O}_2. \]

This is the source of the O₂ we breathe, whose amount is a significant factor in the concerns about deforestation. There are two parts to photosynthesis, which are called the light reaction and the dark reaction.

Light Reaction
The light reaction, which happens in the thylakoid membrane, converts light energy into chemical energy. This chemical reaction must, therefore, take place in the light. Any biologists who is specialized in this sector can tell us that chlorophyll and several other pigments such as beta-carotene are organized in clusters in the thylakoid membrane and are involved in the light reaction. Each of these differently-colored pigments can absorb a slightly different color of light and pass its energy to the central chlorophyll molecule to do photosynthesis. The central part of the chemical structure of a chlorophyll molecule is a porphyrin ring, which consists of several fused rings of carbon and nitrogen with a magnesium ion in the center.
Dark Reaction

Structure of ATP

The energy harvested via the light reaction is stored by forming a chemical called ATP (adenosine triphosphate), which is a compound used by cells for energy storage. This chemical is made of the nucleotide adenine bonded to a ribose sugar, which in turn is bonded to three phosphate groups. This molecule (that) we’re considering is very similar to the building blocks for our DNA.

The dark reaction takes place in the stroma within the chloroplast, and converts CO$_2$ to sugar. This reaction does not directly need light in order to occur, but it does need the products of the light reaction (ATP and another chemical called NADPH). The dark reaction involves a cycle called the Calvin cycle in which CO$_2$ and energy from ATP are used to form sugar. Actually, notice that the first product of photosynthesis is a three-carbon compound called glyceraldehyde 3-phosphate. Almost immediately, two of these join to form a glucose molecule.

Most plants put CO$_2$ directly into the Calvin cycle. Thus the first stable organic compound formed is the glyceraldehyde 3-phosphate. Since that molecule contains three carbon atoms, these plants are called C$_3$ plants. For all plants, hot summer weather increases the amount of water that evaporates from the plant. Plants lessen the amount of water that evaporates by keeping their stomates closed during hot, dry weather. Unfortunately, this means (that) once the CO$_2$ in their leaves reaches a low level, they must stop doing photosynthesis. Even if there is a tiny bit of CO$_2$ left, the enzymes used to grab it and put it into the Calvin cycle just do not have enough CO$_2$ to use. Typically the grass in our yards just turns brown and goes dormant. Some plants like crabgrass, corn, and sugar cane have a special modification to conserve water. These plants capture CO$_2$ in a different way: they do an extra step first, before doing the Calvin cycle. These plants have a special enzyme that can work better, even at very low CO$_2$ levels, to grab CO$_2$ and turn it first into oxaloacetate, which contains four carbons. Thus, these plants are called C$_4$ plants. The oxaloacetate, which the CO$_2$ is released from, is vital: it is this CO$_2$ that is put into the Calvin cycle. This is why crabgrass can stay green and keep growing when all the rest of your grass is dried up and brown.

Note: lawn grass is supposed to go dormant in dry summer weather, and watering it a) wastes a lot of water, b) makes a lot of extra work for the homeowner, and c) wastes energy and causes a lot of pollution due to running the lawnmower all those extra, unnecessary times. Skip the unnecessary water, and you’ll both have a lot more time to enjoy doing other things and create less pollution.

There is yet another strategy (that) we should be interested in so as to cope with very hot, dry, desert weather and conserve water. Some plants (for example, cacti and pineapple) that live in extremely hot, dry areas like deserts, can only safely open their stomates at night when the weather is cool. Thus, there is no chance for them to get the CO$_2$ needed for the dark reaction during the daytime. At night when they can open their stomates and take in CO$_2$, they are these plants which incorporate the CO$_2$ into various organic compounds to store it. In the daytime, when the light reaction is occurring and ATP is available (but the stomates must remain closed), they take the CO$_2$ from these organic compounds and put it into the Calvin cycle. These plants are called CAM plants, which stands for crassulacean acid metabolism after the plant family, Crassulaceae (which includes the garden plant Sedum) where this process was first discovered.
So, defining and non-defining clauses – that is, kernel and non-kernel clauses.

Let's read the following from http://en.wikipedia.org/wiki/Kernel_(linear_algebra):

In linear algebra and functional analysis, the kernel (also null space or nullspace) of a linear map $L : V \to W$ between two vector spaces or two modules $V$ and $W$ is the set of all elements $v$ of $V$ for which $L(v) = 0$. That is

$$
\ker(L) = \{ v \in V : L(v) = 0 \}.
$$

where $0$ denotes the zero vector in $W$. The kernel of $L$ is a linear subspace of the domain $V$.

For a linear map given as a matrix $A$, the kernel is simply the set of solutions to the equation $Ax = 0$, where $x$ and $0$ are understood to be column vectors. The dimension of the null space of $A$ is called the nullity of $A$.

The kernel of an $m \times n$ matrix $A$ with coefficients in a field $K$ (typically the field of the real numbers or of the complex numbers) is the set

$$
N(A) = \text{Null}(A) = \ker(A) = \{ x \in K^n : Ax = 0 \}.
$$

where $0$ denotes the zero vector with $m$ components. The matrix equation $Ax = 0$ is equivalent to a homogeneous system of linear equations:

$$
A\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix}
$$

$$
A\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
$$

From this viewpoint, the null space of $A$ is the same as the solution set to the homogeneous system.

How the null space of a matrix may be computed?

Consider the matrix

$$
A = \begin{bmatrix} 2 & 3 & 5 \\ -4 & 2 & 3 \end{bmatrix}.
$$

The null space of this matrix consists of all vectors $(x, y, z) \in \mathbb{R}^3$ for which

$$
\begin{bmatrix} 2 & 3 & 5 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.
$$

This can be written as a homogeneous system of linear equations involving $x, y,$ and $z$:

$$
2x + 3y + 5z = 0,
$$

$$
-4x + 2y + 3z = 0.
$$

This can be written in matrix form as:

$$
\begin{bmatrix} 2 & 3 & 5 & 0 \\ -4 & 2 & 3 & 0 \end{bmatrix}.
$$

Using Gauss–Jordan elimination, this reduces to:

$$
\begin{bmatrix} 1 & 0 & 1/16 & 0 \\ 0 & 1 & 13/8 & 0 \end{bmatrix}.
$$

Rewriting yields:
\[ x = -\frac{1}{16} z \\
y = -\frac{13}{8} z \]

Now we can write the null space (solution to \( Ax = 0 \)) in terms of \( c \), where \( c \) is scalar:
\[
\begin{bmatrix} x \\ y \\ z \end{bmatrix} = c \begin{bmatrix} -1/16 \\ -13/8 \\ 1 \end{bmatrix}
\]

Since \( c \) is a free variable this can be simplified to
\[
\begin{bmatrix} x \\ y \\ z \end{bmatrix} = c \begin{bmatrix} -1 \\ -26 \\ 16 \end{bmatrix} .
\]

The null space of \( A \) is precisely the set of solutions to these equations (in this case, a line through the origin in \( \mathbb{R}^3 \)).

Let’s give some more examples:

- **If** \( L: \mathbb{R}^n \rightarrow \mathbb{R}^n \), then the kernel of \( L \) is the solution set to a homogeneous system of linear equations. For example, if \( L \) is the operator:
  \[
  L(x_1, x_2, x_3) = (2x_1 + 5x_2 - 3x_3, 4x_1 + 2x_2 + 7x_3)
  \]
  then the kernel of \( L \) is the set of solutions to the equations
  \[
  2x_1 + 5x_2 - 3x_3 = 0 \\
  4x_1 + 2x_2 + 7x_3 = 0.
  \]

- **Let** \( C[0,1] \) denote the vector space of all continuous real-valued functions on the interval \([0,1]\), and define \( L: C[0,1] \rightarrow \mathbb{R} \) by the rule
  \[
  L(f) = f(0.3).
  \]
  Then the kernel of \( L \) consists of all functions \( f \in C[0,1] \) for which \( f(0.3) = 0 \).

- **Let** \( C^\infty(\mathbb{R}) \) be the vector space of all infinitely differentiable functions \( \mathbb{R} \rightarrow \mathbb{R} \), and let \( D: C^\infty(\mathbb{R}) \rightarrow C^\infty(\mathbb{R}) \) be the differentiation operator:
  \[
  D(f) = \frac{df}{dx}.
  \]
  Then the kernel of \( D \) consists of all functions in \( C^\infty(\mathbb{R}) \) whose derivatives are zero, i.e. the set of all constant functions.

- **Let** \( \mathbb{R}^* \) be the direct product of infinitely many copies of \( \mathbb{R} \), and let \( s: \mathbb{R}^* \rightarrow \mathbb{R}^* \) be the shift operator
  \[
  s(x_1, x_2, x_3, x_4, \ldots) = (x_2, x_3, x_4, \ldots)
  \]
  Then the kernel of \( s \) is the one-dimensional subspace consisting of all vectors \((x_1, 0, 0, \ldots)\).

- **If** \( V \) is an inner product space and \( W \) is a subspace, the kernel of the orthogonal projection \( V \rightarrow W \) is the orthogonal complement to \( W \) in \( V \).

The kernel being the solution set to a homogeneous system of linear equations system it obviously deals with a defining clause rather than a non defining one, the main proposition/clause (that is, the output, if one thinks of if clauses, whose kernel is the main clause) **defining** the sentence in order for it to make sense. A set of solution is actually a defining system, from its origin, or an output, that is, the result of any (linguistic) event.

**EXERCISE** Choose any scientific subject and write a text using kernel and non-kernel clauses as far as possible. Then choose any linearly-algebraic kernel, write the corresponding text and show, at any point, any connections with kernel/non-kernel clauses.
PERFORMING SENTENCES ALONG SPACE: VECTORS

Suppose a friend of yours says:

“On Sunday morning I rode a bicycle for 20 km.”

Is the information about displacement complete?

Actually not, for you only know about its entity.
Then you may ask more so that he’s going to add a piece of information:

“On Sunday I rode a bicycle for 20 km along Val d’Adige…”

Well, you may feel better, for he added some information about his direction.

But you are still not satisfied, then you ask more, so that he still adds:

“On Sunday I rode a bicycle for 20 km along Val d’Adige towards Trento.”

The last phrase makes your information actually complete by adding the ‘towards’, which ‘completes’ the direction. Now you are OK—aren’t you?

One more question: can we spatially represent such a sentence?

The answer is yes, of course, and the means is a vector.

As a matter of fact, a sentence conveying movement/displacement, as well as a physical quantity, is a vector when it is necessary to provide it with a magnitude (the entity) and a direction, together with its own ‘towards’ in order to make the information complete.

If you imagine to represent this sentence you may think of a space-oriented straight line, a kind of arrow, whose point, or ‘head’, emphasizes the *towards*, whose length represents its ‘magnitude’, its entity, the 20 km in this case, and whose ‘tail’—actually, the line along which the arrow is lying—represents the direction.

Let’s focus on another example to be found at http://www.physicsclassroom.com/class/vectors/u3l1a.cfm:

"Suppose your teacher tells you ‘a bar of gold is located outside the classroom’. To find it, displace yourself 20 meters.” This statement may provide yourself enough information to pique your interest; yet, there is not enough information included in the statement to find the bag of gold. The displacement required to find the bag of gold has not been fully described. On the other hand, suppose your teacher tells you "A bag of gold is located outside the classroom. To find it, displace yourself from the center of the classroom door 20 meters in a direction 30 degrees to the west of north." This statement now provides a complete description of the displacement vector - it lists both magnitude (20 meters) and direction (30 degrees to the west of north) relative to a reference or starting position (the center of the classroom door). Vector quantities are not fully described unless both magnitude and direction are listed.

Vector quantities are often represented by scaled vector diagrams. Vector diagrams depict a vector by use of an arrow drawn to scale in a specific direction. Vector diagrams were introduced and used in earlier units to depict the forces acting upon an object. Such diagrams are commonly called as free-body diagrams. An example of a scaled vector diagram is shown in the diagram at the right. The vector diagram depicts a displacement vector. Observe that there are several characteristics of this diagram that make it an appropriately drawn vector diagram.

- a scale is clearly listed
- a vector arrow (with arrowhead) is drawn in a specified direction. The vector arrow has a *head* and a *tail*. 

 SCALE: 1 cm = 4 m
• the magnitude and direction of the vector is clearly labeled. In this case, the diagram shows the magnitude is 20 m and the direction is (30 degrees West of North).

• Vectors can be directed due East, due West, due South, and due North. But some vectors are directed northeast (at a 45 degree angle); and some vectors are even directed northeast, yet more northerly than east. Thus, there is a clear need for some form of a convention for identifying the direction of a vector that is not due East, due West, due South, or due North. There are a variety of conventions for describing the direction of any vector. The two conventions that will be discussed and used in this unit are described below:

1. The direction of a vector is often expressed as an angle of rotation of the vector about its "tail" from east, west, north, or south. For example, a vector can be said to have a direction of 40 degrees North of West (meaning a vector pointing West has been rotated 40 degrees towards the northerly direction) of 65 degrees East of South (meaning a vector pointing South has been rotated 65 degrees towards the easterly direction).

2. The direction of a vector is often expressed as a counterclockwise angle of rotation of the vector about its "tail" from due East. Using this convention, a vector with a direction of 30 degrees is a vector that has been rotated 30 degrees in a counterclockwise direction relative to due east. A vector with a direction of 160 degrees is a vector that has been rotated 160 degrees in a counterclockwise direction relative to due east. A vector with a direction of 270 degrees is a vector that has been rotated 270 degrees in a counterclockwise direction relative to due east. This is one of the most common conventions for the direction of a vector and will be utilized throughout this unit.

Two illustrations of the second convention (discussed above) for identifying the direction of a vector are shown below.

Observe in the first example that the vector is said to have a direction of 40 degrees. You can think of this direction as follows: suppose a vector pointing East had its tail pinned down and then the vector was rotated an angle of 40 degrees in the counterclockwise direction. Observe in the second example that the vector is said to have a direction of 240 degrees. This means that the tail of the vector was pinned down and the vector was rotated an angle of 240 degrees in the counterclockwise direction be-
ginning from due east. A rotation of 240 degrees is equivalent to rotating the vector through two quadrants (180 degrees) and then an additional 60 degrees into the third quadrant. (…)

The diagram shows how a velocity vector with a magnitude of 50 m/s and a direction of 60 degrees above the horizontal may be resolved into two components. The diagram shows that the vector is first drawn to scale in the indicated direction; a parallelogram is sketched about the vector; the components are labeled on the diagram; and the result of measuring the length of the vector components and converting to m/s using the scale.

**SCALE: 1 cm = 10 m/s**

![Diagram showing vector components](image)

Trigonometric functions relate the ratio of the lengths of the sides of a right triangle to the measure of an acute angle within the right triangle. As such, trigonometric functions can be used to determine the length of the sides of a right triangle if an angle measure and the length of one side are known. Let’s say we want to know the components of the force acting upon Fido. As the 60-Newton tension force acts upward and rightward on Fido at an angle of 40 degrees, the components of this force can be determined using trigonometric functions.
In conclusion, a vector directed in two dimensions has two components - that is, an influence in two separate directions. The amount of influence in a given direction can be determined using methods of vector resolution. Two methods of vector resolution can be used: - a graphical method (parallelogram method) and a trigonometric method”.

Consequently if we want to represent our first sentence in 2D we should use a diagram like that

\[
\sin 40^\circ = \frac{F_{\text{vert}}}{60 \text{ N}} \quad \cos 40^\circ = \frac{F_{\text{horiz}}}{60 \text{ N}}
\]

\[
F_{\text{vert}} = 60 \text{ N} \times \sin 40^\circ \quad F_{\text{horiz}} = 60 \text{ N} \times \cos 40^\circ
\]

\[
F_{\text{vert}} = 38.6 \text{ N} \quad F_{\text{horiz}} = 45.9 \text{ N}
\]

where the magnitude r performs the length, the 20 km, the direction must be consistent with the angles and the slope is the ratio rise/run. In a right triangle we know that the \( \sin \theta = \text{opposite side/hypotenuse}, \)
the \( \cos \theta = \text{adjacent side/hypotenuse}, \) while the \( \tan \theta = \text{slope= opposite/adjacent}. \) We now know everything we need to know: where your friend is exactly directed, how much he rises, how much he runs, the direction, that is, the angle along which he moves. Fair enough! We can be precise enough.
But otherwise, we move in 3-D, we do not deal with a movement just ‘along’ something: we need to say ‘over’! So, we need to find out an appropriate system that be consistent with that ‘over’. Here we go:

![Diagram of spherical coordinates](image)

\[
x = \rho \sin \theta \cos \phi \\
y = \rho \sin \theta \sin \phi \\
z = \rho \cos \theta
\]

Not only length, then, but also width and depth. It takes three numbers to identify a person’s location at any given time: longitude, latitude and altitude/depth. These dimensions allow us to move in any direction that we wish. The angle \( \phi \) allows us to specify all hidden directions whether or not it’s us who moves or it’s the system’s (relative) motion. With a reference system like that your friend may add sentences so as to form a text like the following:

On Sunday, while I was riding a bicycle for 20 km along Val d’Adige towards Trento I could admire wonderful green leaves of grass on my right side, while on the left there were millions of trees. Majestic mountains spread over my head, while the sun was bursting right out of the sky above them and a sparkling river was flowing under my feet …

The angles’ rotations and values give us at any time the direction of the point \( P \), your friend, and allow him to talk space into real.