Syllabus of the course "Quantum Mechanics"
Instructor: Dr. Giulia Maria de Divitiis
Fall 2014 – Academic Year 2014-2015

1. Introduction. Quantum states, observables and time evolution. Schroedinger, Heisenberg and the interaction pictures. The Dyson series. Schwarz inequality and the generalized uncertainty principle.

2. The Density matrix and its properties. Pure and mixed states. Time evolution of the density operator: the von Neumann equation. Indicators of purity the ensemble: ρ^2 and the Von Neumann entropy.
   Examples: The probability distribution at the thermal equilibrium, the case of spin 1/2 particles with partial polarization.

3. The coherent states of the harmonic oscillator. Heisenberg operators. Expansion in the energy eigenstate. Minimum uncertainty wave packets.

4. The WKB approximation. Criterions of validity. Airy functions. The connection rules. Bohr-Sommerfeld energy quantization condition. Tunneling through a potential barrier.

5. The propagator as a transition amplitude, general properties. Propagator as a Green's function. The Feynman's path integral representation. Examples: the free particle, the harmonic oscillator.
   The Aharonov-Bohm effect. The magnetic monopoles and the charge quantization.

6. The angular momentum. Rotation matrices and the Schwinger method. Spherical tensors. The Wigner–Eckart theorem.

7. The central potentials. The two body problem: separation of the center of mass and the relative coordinates. The radial equation: the hydrogen atom ($E < 0$) and the free particle($E > 0$): the spherical Bessel and Neumann functions.
   Partial wave decomposition of plane waves.

8. Basic concepts of scattering processes, definition of differential cross section, total cross section , scattering amplitude.
   The optical theorem.
   The Lippmann-Schwinger integral equation.
   The Born series. The first Born approximation. Example: the Yukawa potential.

9. Partial wave analysis. An example, the hard sphere.
   Partial wave scattering amplitude. Partial cross section. Unitarity bound.
   Check of the optical theorem. Scattering at low energies, the scattering lengths.
   Resonances, Argand diagrams, Breit-Wigner cross section.

10. Elements of special relativity.
    Contravariant and covariant four-vectors. Scalar product, metric tensor. The Lorentz group: boosts, rotations, time reversal and space inversions. Classifications of the Lorentz transformations. Infinitesimal transformations and generators.

11. The Klein–Gordon equation. The free solutions. The four current density. Not positive definite probability density and negative energies. Lagrangian formulation for relativistic fields and field equation. The Klein–Gordon lagrangian. Noether's theorem and charges. The minimal substitution and the covariant derivative.

12. The Dirac equation. The α, β matrices and the Dirac algebra. The conserved four current density. The free solutions labelled with the sign of energy and with the helicity. The covariant formulation of the Dirac equation and the γ matrices. The Dirac lagrangian. The non relativistic limit and the Pauli equation.

13. Lorentz covariance of the Dirac equation. Explicit construction of a proper Lorentz transformation and of parity. Transformation laws of bilinear densities.