A.A. 2014/2015

Prof. Rubik Poghosyan

Lecture 1.

Basic notions of set theory and topology: Metric spaces; open, closed, compact sets, Heine-Borel theorem; Weierstrass theorem.

Lecture 2.

Complex numbers; functions of complex variable; roots of polynomials; extended complex plane, Riemann sphere, stereographic projection.

Lecture 3.

Power series, radius of convergence, elementary functions: exp, log, sine, cos, arcsine; single valued and multivalued functions; bilinear (fractional) transformations and their properties, cross ratio.

Lecture 4.

Derivative, analytic functions, Cauchy-Riemann equation, harmonic functions, geometric meaning of the derivative.

Lecture 5.

Complex integration; Cauchy's theorem (simply connected and non-simply connected cases), Morera's theorem, Cauchy's integral formula, formula for derivatives.

Lecture 6.

Reciprocal of a power series; Bernoulli numbers, recurrent relation for the Bernoulli numbers; expansion of cot and coth.

Lecture 7.

Mean value theorem; maximum modulus theorem; Cauchy's inequalities; Liouville's theorem; Taylor series; Laurent expansion.

Lecture 8.

Singularities of an analytic function, isolated singularities and their classification (removable singularity, pole, essential singularity); residue theorem; singularities at infinity; Weierstrass and Picard theorems

Lecture 9.

Meromorphic functions; index theorem; Rouche's theorem; invers function theorem; Lagrange formula.

Lecture 10.

Application of complex function methods for computation of definite integrals; Jordan lemma.

Lecture 11.

Meromorphic functions with prescribed principal parts; Mittag-Leffler representation of meromorphic functions; Lagrange formula for the rational functions;

Lecture 12.

Entire functions with prescribed zeros; Weierstrass representation of an entire function as an infinite product.

Lecture 13.

Mittag-Leffler series and Weierstrass infinite product representation using contour integration (1/sin z, cotan z, sin z); Riemann Zeta-function at even integer values.

Lecture 14.

Asymptotic methods: asymptotic series, exponential integral Ei(z); Euler-Maclaurin formula (by contour integral), the case of finite sum and the remainder term.

Lecture 15.

Asymptotic methods: application for the factorial, computation of the constant A using Wallis formula; Laplace's method; method of steepest descent.

Lecture 16.

Gamma function: defining integral representation, Hankel representation, Euler infinite product.

Lecture 17.

Euler-Mascheroni constant, Weierstrass product representation of Gamma function, main identities.

Lecture 18.

Digamma function (logarithmic derivative of Gamma): Mittag-Leffler type expansion, difference relation, identities, integral representation;

Lecture 19.

Euler Beta function, expression through Gamma function, Pochhammer's integral.

Lecture 20.

Riemann Zeta function: generalized Zeta function (definition and integral representation), Hankel like representation, Hurwitz formula, reflection relation for the Riemann Zeta function, Relation with prime numbers, Euler Euler product, Riemann hypothesis.

Lecture 21.

Linear differential equations: 2^{nd} order homogeneous equation, Power series solution; Uniqueness and analyticity of solution around ordinary points; Wronskian; Solution of the inhomogeneous equation.

Lecture 22.

Solution around regular singular points, indicial equation, existence of solutions and their analytic properties; the case when indicial roots differ by an integer; Behavior of solutions, monodromy

Lecture 23.

Equation with three regular singular points (Riemann's P-equation), the case when one of the regular singular points is at the infinity; Transformations of P-equation: bilinear (fractional) transformation of the dependent variable, shift of indices;

Lecture 24.

Hyper-geometric equation; Reduction of Riemann's P-equation to the Hyper-geometric equation; Hyper-geometric function; Kumar's 24 solutions in terms of Hyper-geometric functions; Properties of Hypergeometric function: relations among contiguous functions, derivatives;

Lecture 25.

Contour integral representations of the Hyper-geometric function: Euler's representation, Barnes' contour integral; Derivations of various identities (which provide analytic continuation) using integral representations;

Lecture 26.

Laplace's equation in arbitrary (curved) coordinates; Laplace's equation in three dimensions in spherical coordinates: separation of variables; Legendre's associated equation; The special case referred as Legendre's equation: solutions in terms of Hyper-geometric functions;

Lecture 27.

The regular solutions of Legedre's equation: Legendre's polynomials; Quantum mechanical theory of angular momentum and its relation to the Legendre's equation; Spherical functions; Rodriguez' formula for the Legendre's polynomials; Generating function of the Legendre's polynomials;

Lecture 28.

Integral transforms; Fourier transform; Fourier's theorem, Dirac's delta-function: various representations and properties; Multi-dimensional Fourier-transform;

Lecture 29.

Fourier transform of a derivative; Convolution theorem, Perceval's relation; Applications of the Fourier transform: Heat flow equation, wave equation;

Lecture 30.

Method of Green's function: Green function of the Poisson equation;